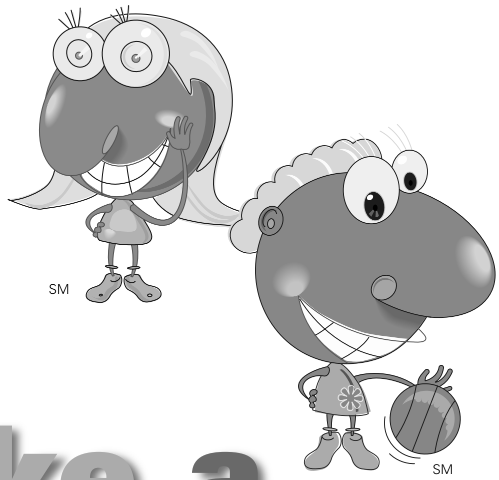
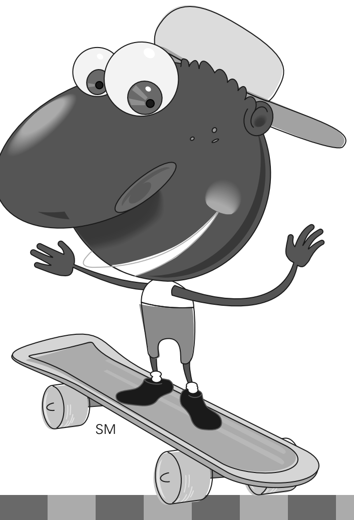
Figure This! Math Challenges for Families℠



Take a Challenge!

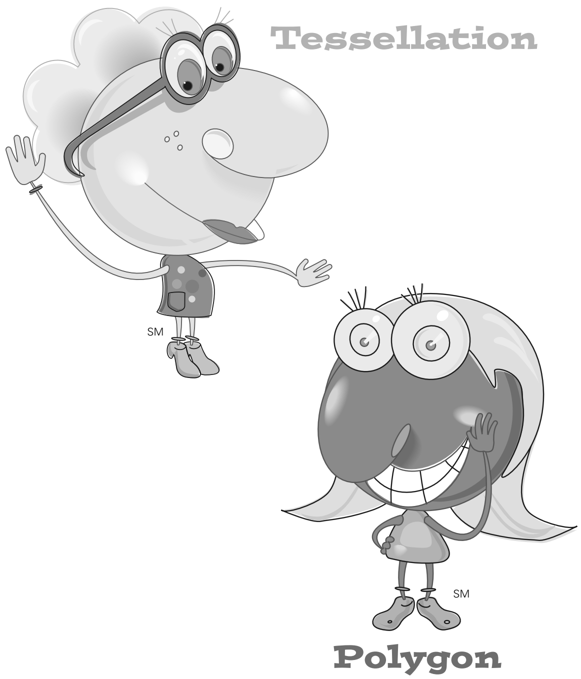
Set II: Challenges 17 - 32

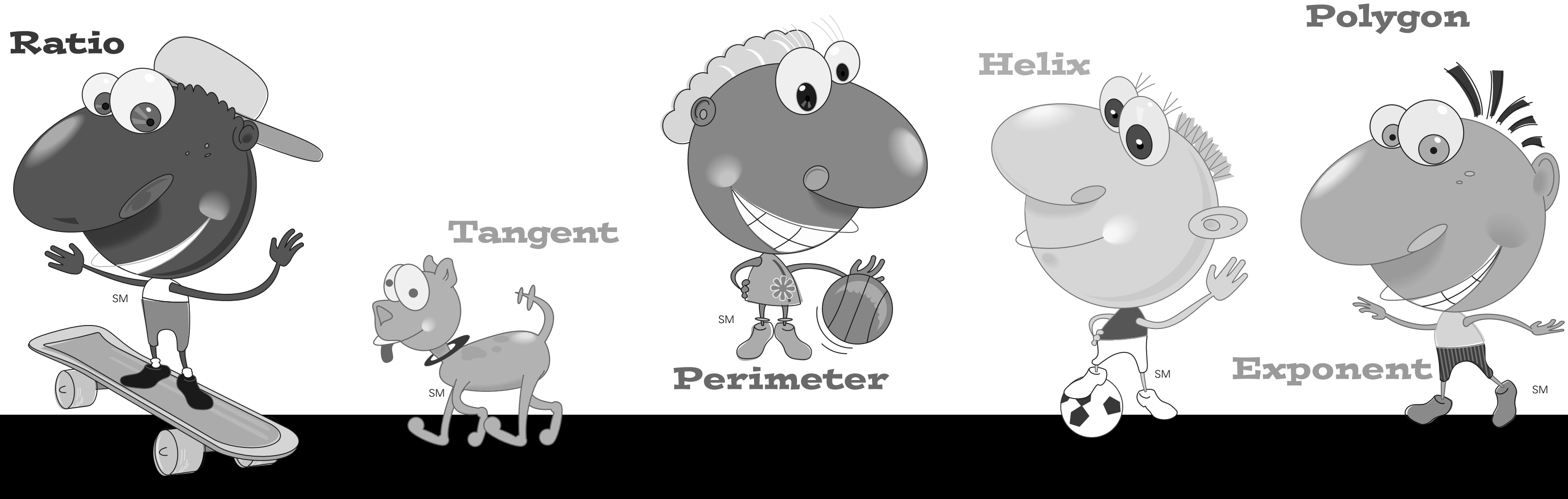


FigureThis! Math Challenges for Families

Thank you for your interest in the **FigureThis! Math Challenges for Families.** Enclosed please find Challenges 17 – 32. For information about other challenges, go to www.figurethis.org.

The **Figure This!** Challenges are family-friendly mathematics that demonstrate what middle-school students should be learning and emphasize the importance of high-quality math education for all students. This campaign was developed by the National Action Council for Minorities in Engineering, the National Council of Teachers of Mathematics, and Widmeyer Communications, through a grant from The National Science Foundation and the US Department of Education.





We encourage you to visit our website at **www.figurethis.org** where you can find these and other challenges, along with additional information, math resources, and tips for parents.

Figure This!

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Math Challenge #17

PERPLEXING PERCENTAGES

Figure This! Math Challenges for Families

So how much does it cost?

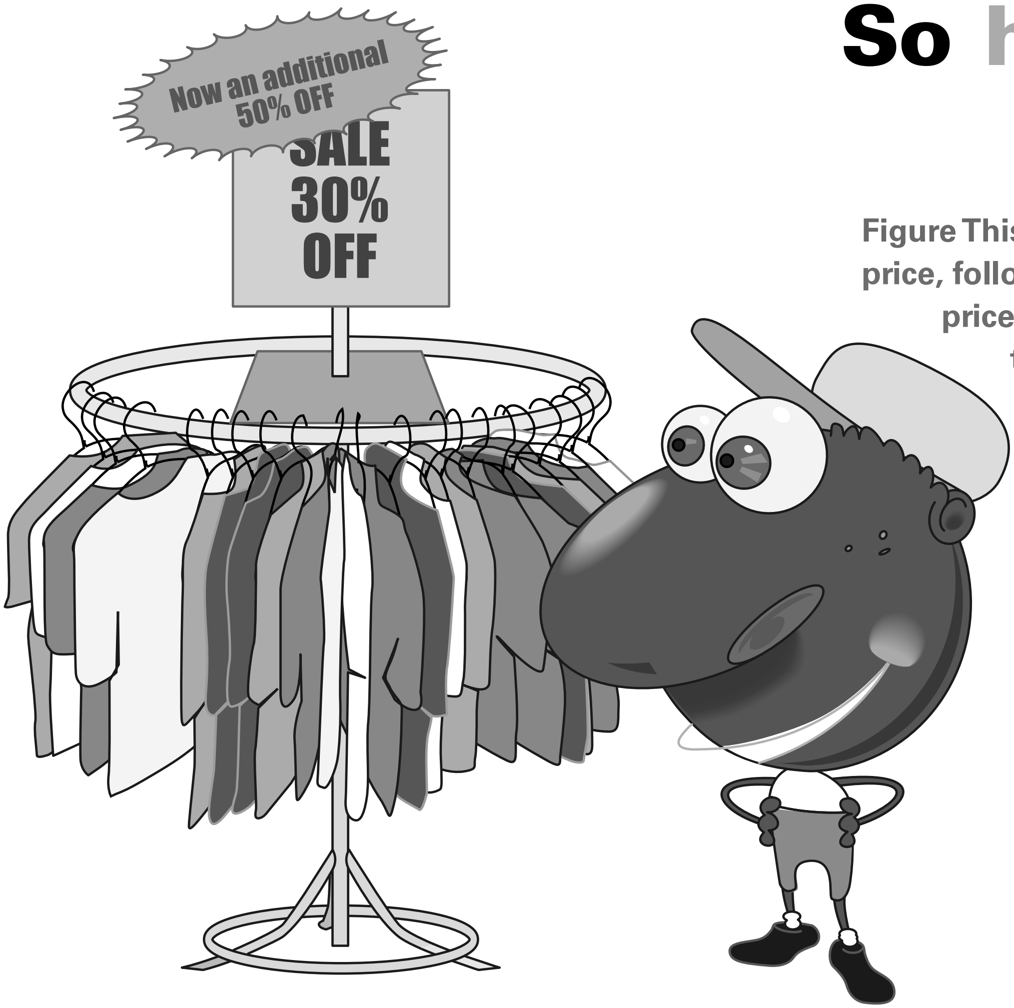


Figure This! Is a discount of 30% off the original price, followed by a discount of 50% off the sale price, the same as a discount of 80% from the original price?

Hint: What would a $100 item cost after these discounts?

Understanding percentages is critical in many everyday and business decisions. Survey results, medical reports, weather information, and interest rates all involve percentages.

Answer:
No. The discount is not 80%.

Figure This!

Get Started:

Choose a price for an item, say $100 as suggested in the hint. Calculate what the sale price would be after a 30% discount. Then find out how much the item would cost at 50% off the sale price.

Complete Solution:

- If an item originally costs $100, the tables below show the different final costs. They are not the same.

Original Price	30% Off	Cost on Sale	50% Off Sale Price	Final Cost
$100	30% • $100 = $30	$100 - $30 = $70	50% • $70 = $35	$70 - $35 = $35

Original Price	80% Off	Final Cost
$100	80% • $100 = $80	$100 - $80 = $20

- For the item on sale at 30% off, you would need to pay 70% of the price. So an additional discount of 50% off the sale price would bring the price to 35% (that is, 50% • 70%) of the original price. Thus, a $100 item would cost $35 after both discounts. An 80% off sale means that you pay 100% – 80%, or 20% of the original cost of the item. Thus, an item that originally cost $100 on sale at 80% off costs 20% • $100 or $20. The costs are not the same.
- You can generalize the problem. If P is the original price of an item, with the two discounts, one of 30% followed by another of 50%, you would pay 0.50 • (0.70 • P) or 0.35P, which is not the same as 0.2P.

Try This:

- Look at some of the discounts offered in newspaper or magazine ads. Find examples that use multiple discounts and calculate the actual cost per item.

Additional Challenges:

1. Would you rather become 50% richer and then 50% poorer, or become 50% poorer and then 50% richer?
2. The original price of a washing machine is $500. On the first day of each month, the store will reduce its price by 10% of the previous price. How long will it take before the sale price is half the original price?
3. An ad in a clothing store reads, "Clearance: 60% to 75% off when you take an extra 50% off the previous sale price." The previous sale price on a pair of jeans was $24.99, down from an original $29.99. Is the ad correct for this item?

Things to Think About:

- A discount of 50% is the same as a half-price sale.
- A discount of 25% is the same as paying 75% of the price.
- A cost of 10% more than a price is 110% of the listed price.
- In what situations are percentages more useful than fractions?

Did You Know That?

- The word percent comes from the Latin *per centum*, meaning "per 100."
- Pressing the percent key on some calculators changes the percentage to a decimal.

Resources:

Books:

- Paulos, John Allen. *Innumeracy: Mathematical Illiteracy and Its Consequences*. New York: Hill and Wang, 1988.
- Paulos, John Allen. *A Mathematician Reads the Newspaper*. New York: Basic Books, 1995.

Answers to Additional Challenges:

(1.) :
The result is the same.

(2.) :
In the seventh month, the cost will be less than half the original price.

(3.) :
The final cost is $24.99 – (0.5 • $24.99), or $12.50. The total discount from the original price is $29.99 – $12.50 or $17.49. Since $17.49 is about 58% of $29.99, the ad is not correct.

Math Challenge
#18
TURN TERN

Figure This! Math Challenges for Families

Are we there yet?

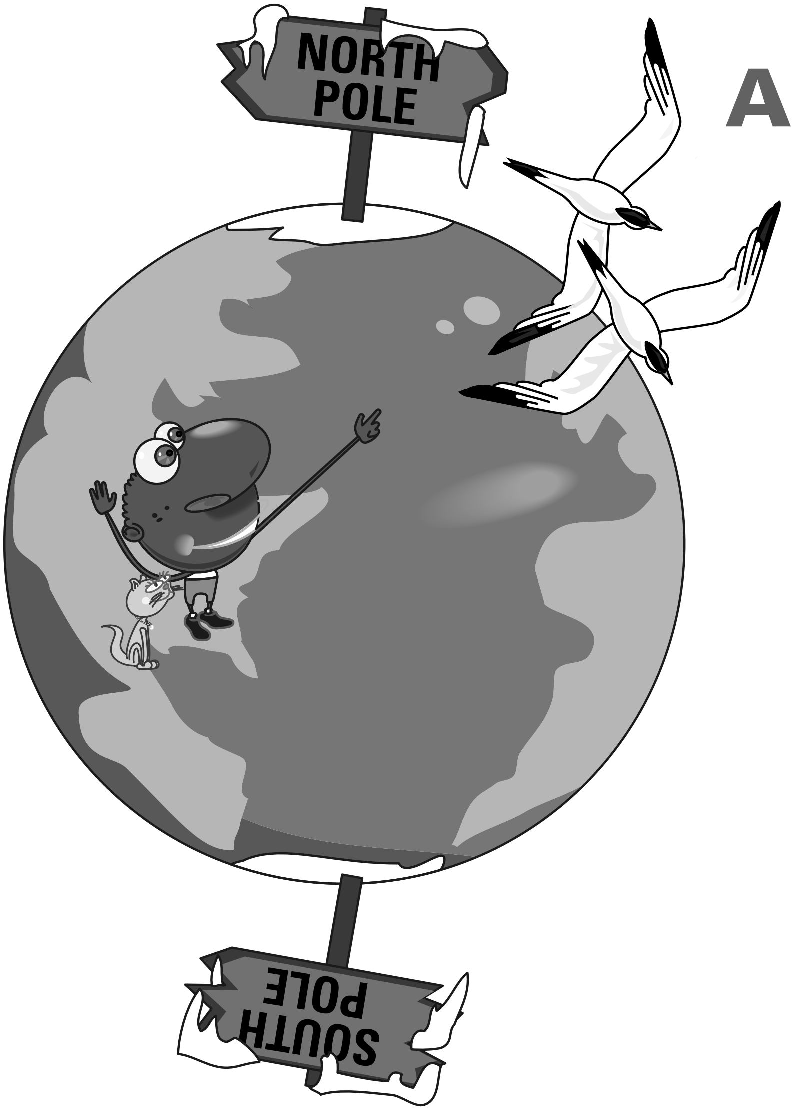


Figure This! **Every year, Arctic terns fly from the Arctic to the Antarctic and back, a distance of about 9000 miles each way. Suppose the birds fly at an average speed of 25 miles per hour for 12 hours a day. How many days of flying would be necessary to make the roundtrip?**

Hint: How many miles would a tern fly in an average day?

The distance traveled by a moving object can be found using its rate and time. Calculating distance is important for railroad companies, airlines, and trucking firms, as well as family travelers on vacation.

Answer:
Flying at 25 mph for 12 hours per day, a tern could make the roundtrip journey in about 60 days.

Figure This!

Get Started:

Find the number of miles a tern flies in one day; then find the number of days required to fly each way.

Complete Solution:

At 25 mph, a tern would travel 100 miles in 4 hours. Flying 12 hours in a day, the bird would cover 300 miles. Therefore, the one-way journey of 9000 miles would require 30 days.

25 mph x 12 hr/day = 300 miles/day
9000 miles ÷ 300 miles/day = 30 days

The roundtrip would take twice that, or about 60 days.

Try This:

- Determine how long it takes you to walk a mile. Estimate how many miles you could walk in a day without getting too tired. Could you walk across the United States in one year?
- Find out about other animal migrations, such as those of robins, swallows, or whales.

Additional Challenges:

1. Monarch butterflies migrate from Canada to Mexico, a distance of about 2500 miles. If it takes them about one month, about how many miles do they fly per day? If they fly for 12 hours per day, what is their average speed in miles per hour?
2. The distance around the earth at the equator is about 25,000 miles. How long would it take an Arctic tern like the ones described in the challenge to fly this distance?
3. The fastest bird is thought to be the peregrine falcon. Its top speed is at least 124 mph. The slowest birds are the American and Eurasian woodcock, which can fly at only 5 mph without stalling. Compare the time it would take a falcon and a woodcock to fly across the United States (approximately 2500 miles).

Things to Think About:

- In 1850, wagon trains usually took from 4 to 6 weeks to travel from Missouri to California, about 2000 miles.
- Some swallows return to San Juan Capistrano, California, at about the same time every year. How do the birds know when and where to go?
- Can a butterfly go faster than you can run?
- How fast is a breeze?

Did You Know That?

- The exact details of the Arctic tern's migration, as with most bird migrations, are unknown. The route is not necessarily direct, and the birds make stops along the way.
- Naturalists catch birds, place identification bands on their legs, then release them. If these birds are caught again, the bands help provide a record of where the birds came from.
- An Arctic tern banded in the Arctic Ocean was captured again three months later—11,000 miles away.
- Observers of the Arctic tern report that they can be found for three to four months in the Arctic region, and for 3 to 4 months in the Antarctic region.
- The blackpole warbler cannot swim, yet it migrates over the open ocean from New England to South America, a nonstop trip of over 4000 miles.
- Birds have been seen at elevations of 30,000 feet — higher than Mount Everest. One bird crashed into an airliner flying at 37,000 feet.
- Carrier pigeons have been clocked at speeds of 35 mph.

Resources:

Book:

- *The Guinness Book of Records*. New York: Guinness Book of Records, 1999.

Website:

- **www.mmm.com/front/monarch/index.html**
- **www.schoolnet.ca/collections/arctic/species/birdsgen.htm**
- **www.randomhouse.com./features/audubon/nas/**

Answers to Additional Challenges:

(1.)

The butterflies travel an average of 83 miles per day. If the fly only 12 hours per day, their average speed would be approximately 7 mph.

(2.)

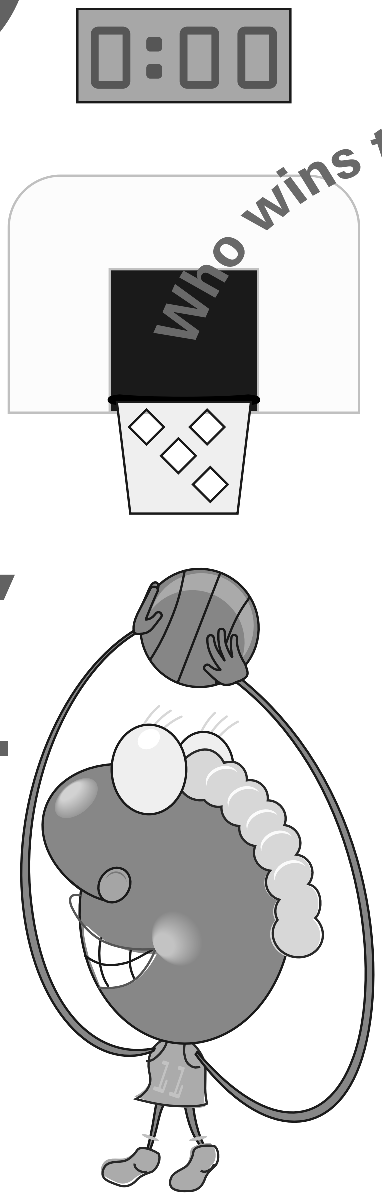
At the speed given in the challenge, about 83 days.

(3.)

If a falcon could maintain its top speed of 124 mph, it could fly the distance in about 20 hours. At 5 mph, a woodcock would need about 500 hours of flying time. The difference is 480 hours, or 20 full days.

Math Challenge #19 TWO POINTS!

FigureThis! Math Challenges for Families



Who wins the game???

she shoots, and... she...

Figure This! Your team is down by one point. Your teammate, who makes free throws about three-fourths of the time, is at the free throw line. She gets a second shot if she makes the first one. Each free throw that she makes is worth 1 point. If there is no time left, what are the chances you win the game with no overtime?

Hint: Tossing two different coins can be used to represent making or missing a shot. What are the outcomes when you toss two coins? How can you use this to model the free throws?

Probability is a measure of chance. Applications of probability are found in genetics, the insurance industry, lotteries, and medical testing.

Answer:
The probability that your team wins is 9/16 or about 56% of the time.

Figure This!

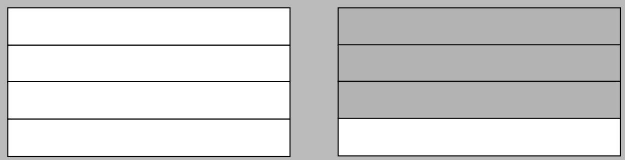
Get Started:

Toss two coins as suggested in the hint. Let getting two heads or a head and a tail represent making the shot; let getting two tails represent missing the shot. Toss the coins once. If you have two tails, she missed the free throw and the game is over. If you have at least one head, she made a point and the game is tied; she gets a second shot. Toss the coins again. Did your team win, or is the game still tied? Do this experiment about 50 times to predict whether your team wins or not.

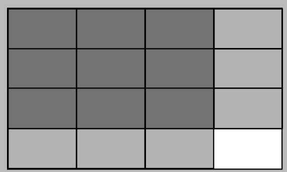
Complete Solution:

There are many ways to answer the question.

- Make a rectangular diagram with four rows of the same size. Shade three of the four rows to represent making the first shot.

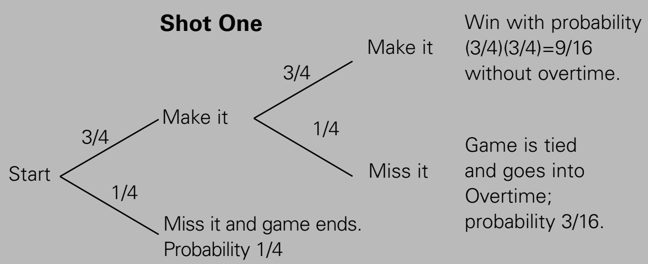


Make four columns of the same size and shade three of them to represent making the second shot. There should now be 16 cells in the grid. The 9 cells that are shaded twice represent success on both shots which means your team wins without any overtime play.



Since 9 of the 16 equally likely outcomes represent wins, the probability of winning is 9/16.

- A different strategy is to draw a tree diagram labeled with all outcomes and their probabilities for each shot. The probability of winning is found by multiplying the probabilities on the appropriate branches of the tree. If your teammate makes the first shot 3/4 of the time, then 3/4 of those times that she makes the first shot, she will make the second shot; that is (3/4)(3/4) = 9/16. In this case, the probability of winning without overtime is 9/16.



[Multiplying probabilities here is correct only if the two shots are independent events. Assume that they are.]

Try This:

Find the probability of getting two heads when you toss a coin twice even if you don't have a penny! You can model what happens with a piece of rectangular paper. Fold a sheet of paper in half lengthwise and mark one half "Heads" to represent obtaining a head on the first toss. Mark the other half "Tails." Fold the paper again. The creases in the paper should now divide it into four equal parts. One part should be labeled "Heads-Heads;" one "Heads-Tails;" one "Tails-Heads;" and one "Tails-Tails." The probability of each outcome is represented by the portion of the paper it occupies.

Additional Challenges:

1. What is the most likely outcome for your team in the game described in the Challenge?
2. How would you change your answer if the player's free-throw percentage was 60%?
3. What free-throw percentage should your teammate have to give your team a 50% chance of winning in this situation?

Things to Think About:

- Near the end of a close game, do National Basketball Association (NBA) players use free-throw percentages to decide whom to foul?
- Probability may be estimated based on the overall pattern after many, many events.
- The probability of an absolutely sure bet is 100% or 1. For any probability less than that, knowing that something "can" happen is no guarantee that it will happen.

Did You Know That?

- When you toss a coin eight times, the chance of getting four heads in a row and then four tails in a row is the same as getting a head and then alternating tails and heads.
- Computer or calculator simulations can be used to estimate the probability of events.
- Probability was invented to decide how to divide the winnings fairly when the players had to leave a game before it was over.

Resources:

Books:

- Gnanadesikan, Mrudulla, Richard L. Scheaffer, and Jim Swift. *The Art and Techniques of Simulation*. White Plains, NY: Dale Seymour Publications, 1987.
- Hopfensperger, P., H. Kranendonk, and R. Scheaffer. *Data-Driven Mathematics: Probability through Data*. White Plains, NY: Dale Seymour Publications, 1999.
- Newman, C. M., T. E. Obremski, and R. L. Scheaffer. *Exploring Probability*. White Plains, NY: Dale Seymour Publications, 1987.

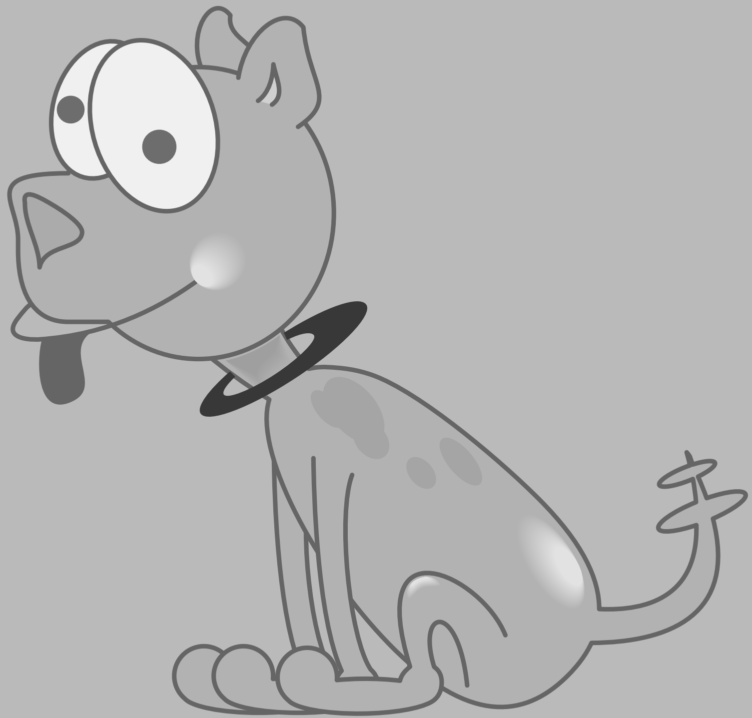
Answers to Additional Challenges:

(1) :
Before she begins to shoot, the probability of winning without overtime is 9/16. The probability of a tie-game is 3/16 and in this case, the game must continue to determine the winner. The probability of losing without overtime is 4/16 or 1/4. The most likely outcome is winning the game without overtime.

(2) :
The most likely outcome is losing the game. The probability of losing without overtime is 40%.

(3) :
To have a 50% chance of winning without overtime, the free-throw percentage would be $\sqrt{0.50}$ or about 0.71 = 71%.

Tangent



Math Challenge #20

BASKETBALL PICKS

Figure This!

Math Challenges for Families

WHO PLAYED THE RAPTORS???

Figure This! There are four basketball games Saturday night. Three sportswriters predicted the winners in the Saturday morning paper.

- Perimeter picks the Raptors, Pacers, Magic, and 76ers.
- Exponent picks the Hawks, Pistons, Magic, and Raptors.
- Helix picks the Heat, Pacers, Pistons, and Raptors.
- No one picks the Bucks.

WHO PLAYED WHOM?

Hint: Teams chosen by the same sportswriter did not play each other.

Organizing and analyzing information to make logical decisions are important skills in many professions. Company managers, doctors, and scientists all use these skills.

Answer:

Raptors played Bucks, Pacers played Hawks, Magic played Heat, and Pistons played 76ers.

Figure This!

Get Started:

Since each game can only have one winner, the four winners picked by each sportswriter are not playing each other. One way to organize the rest of the information is to make a chart like the one shown below. There are eight teams involved in the four games. Each X in the chart shows a match-up that can be ruled out using Perimeter's picks. What match-ups can be ruled out using Exponent's picks?

	Raptors	Pacers	Magic	76ers	Hawks	Pistons	Heat	Bucks
Raptors	X	X	X	X				
Pacers	X	X	X	X				
Magic	X	X	X	X				
76ers	X	X	X	X				
Hawks								
Pistons								
Heat								
Bucks								

Complete Solution:

- One way to do this problem is to use each writer's picks to complete the chart described in the Get Started section. If a row or column has an X in all spaces but one, the unmarked space shows two teams that play each other.

	Raptors	Pacers	Magic	76ers	Hawks	Pistons	Heat	Bucks
Raptors	X	X	X	X	X	X	X	
Pacers	X	X	X	X		X	X	
Magic	X	X	X	X	X	X		
76ers	X	X	X	X				
Hawks	X		X		X	X		
Pistons	X	X	X		X	X	X	
Heat	X	X				X	X	
Bucks								X

This method shows that the only possible match-up for the Raptors is the Bucks. Put an O in the cells for a Raptors–Bucks game. Fill in the rest of the Bucks' row and column with Xs. The chart now shows that the Pacers must play the Hawks. Continuing to reason in this way, the Magic played the Heat, and the 76ers played the Pistons.

	Raptors	Pacers	Magic	76ers	Hawks	Pistons	Heat	Bucks
Raptors	X	X	X	X	X	X	X	O
Pacers	X	X	X	X	O	X	X	X
Magic	X	X	X	X	X	X	O	X
76ers	X	X	X	X	X	O	X	X
Hawks	X	O	X	X	X	X	X	X
Pistons	X	X	X	O	X	X	X	X
Heat	X	X	O	X	X	X	X	X
Bucks	O	X	X	X	X	X	X	X

- Another way to organize the information is to make a list of the teams chosen by Perimeter on the left side. List the other four teams along the top.

	Hawks	Pistons	Heat	Bucks
Raptors	____	____	____	____
Pacers	____	____	____	____
Magic	____	____	____	____
76ers	____	____	____	____

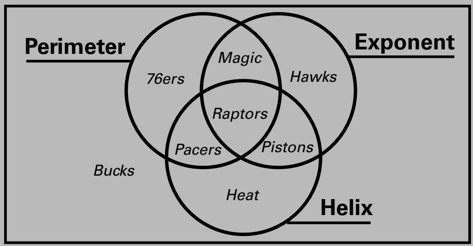
Exponent picked the Hawks, Pistons, Magic, and Raptors. So the Hawks did not play the Pistons, Magic, or Raptors. Mark an X in the blanks pairing the Hawks with the Raptors and Magic. The Pistons cannot play the Raptors or Magic. Helix's picks are the Heat, Pacers, Pistons, and Raptors. So the Heat cannot play the Raptors or Pacers.

	Hawks	Pistons	Heat	Bucks
Raptors	X	X	X	____
Pacers	____	X	X	____
Magic	X	X	____	____
76ers	____	____	____	____

This shows that the Raptors must play the Bucks and the 76ers have to play the Pistons. Fill in the rest of the Bucks column with Xs because no other team can play them. This leaves one empty cell in the Magic row, so the Heat plays the Magic. Finally, the Pacers play the Hawks.

	Hawks	Pistons	Heat	Bucks
Raptors	X	X	X	O
Pacers	O	X	X	X
Magic	X	X	O	X
76ers	X	O	X	X

- A different method to solve this problem is using an arrangement of circles called a Venn diagram. Each sportswriter's picks can be thought of as a set, and two teams that are in the same set cannot play each other. Using the information given in the Challenge, you can draw a Venn diagram like the one below.



All three sportswriters picked the Raptors, so they are included in all three circles. Since the Bucks are the only team outside these three circles, the Raptors must play the Bucks. Perimeter and Helix selected the Pacers. The only remaining team not selected by both of these writers is the Hawks, since it is not in either of those circles. So the Pacers must play the Hawks. Similarly, the 76ers play the Pistons and the Magic play the Heat.

Try This:

- Check the sports page in your local paper and see if it publishes picks for sporting events. Looking only at the picks, can you determine which teams will play each other on a given day?

- Lewis Carroll—the author of *Alice in Wonderland* and other favorites—wrote the following logical arguments in which the last statement is a conclusion based upon the first two.

Every eagle can fly;	All wasps are unfriendly.
Some pigs cannot fly.	No puppies are unfriendly.
Some pigs are not eagles.	Puppies are not wasps.

Make up some logical arguments of your own like the ones Carroll wrote. Try your arguments on a friend to see if they believe them.

Additional Challenges:

1. Justin, Aneisha, Steven, and Trish each brought a different pet to their community pet show. Using the following clues, match each pet—a dog, a cat, a horse, and a snake—with its owner.
   - Steven is the brother of the person who owns the snake.
   - Aneisha and Trisha do not like animals that bark.
   - Justin prefers reptiles over mammals.
   - Trisha hopes to ride her pet in an upcoming parade.

2. An automobile dealer asked 100 customers if they liked the colors green, white, and black. The results of this survey are shown below.
   - 55 said they liked green
   - 47 said they liked white
   - 15 said they liked both green and white but not black
   - 5 said they liked both white and black but not green
   - 20 said they liked both green and black but not white
   - 10 said they liked green, white, and black
   - 12 said they liked only black

   How many customers did not like green, white, or black?

Things to Think About:

- How do census takers sort the thousands of pieces of information needed to report trends in populations, jobs, and salaries?
- In this challenge, there was only one schedule possible, given the sportswriters' picks. Would this always be true?
- How do sportswriters conclude that a team has clinched a title before the season is finished?

Did You Know That?

- Englishman John Venn (1834–1923) developed the Venn diagram to represent sets and their unions and intersections.
- Blood typing can be represented with Venn diagrams.
- Lewis Carroll's *Alice in Wonderland* contains many logic puzzles.
- Sherlock Holmes, created by Arthur Conan Doyle, used logical reasoning to solve mysteries.

Resources:
Books:

- Carroll, Lewis. *Alice's Adventures in Wonderland.* Cambridge, MA: Candlewick Press, 1999.
- Carroll, Lewis. *In The World of Mathematics,* Vol. 4. Newman, James R., ed. New York: Simon and Schuster, 1956. pp. 2397-2415.
- Doyle, Arthur Conan. *The Complete Sherlock Holmes*. New York: Doubleday and Co., 1953.
- Dudeney, H. E. *Amusements in Mathematics.*New York: Dover Publications, 1970.
- Moore, Rosalind, ed. *The Dell Book of Logic Problems*. New York: Dell Publishing Co., 1984.
- Smullyan, Raymond. *The Lady and the Tiger*. New York: Alfred E. Knopf, 1982.
- Smullyan, Raymond. *What Is the Name of This Book?* Englewood Cliffs, NJ: Prentice-Hall, 1978.

Website:

www.inficad.com/~ecollins/logic-prob.htm

Answers to Additional Challenges:

(1.)
Justin owns the snake, Aneisha owns the cat, Steven owns the dog, and Trisha owns the horse

(2.)
11 customers.

Math Challenge #21

BODY MASS

Figure This!
Math Challenges for Families

what's your INDEX???

Figure This! Some doctors use body-mass index as an indicator of health risk. According to *The Old Farmer's Almanac 2000,* body-mass index (BMI) can be found using the formula:

$$\text{BMI} = \frac{(W \times 705) \div H}{H}$$

where H is height in inches and W is weight in pounds. According to the *Almanac,* an index greater than 27 or less than 19 indicates an increased risk for health problems. Helix is 5 feet, 2 inches tall and weighs 110 pounds.

Is his health at risk?

Hint: Convert Helix's height to inches, then use the formula.

Using and understanding formulas is a critical skill in almost every field, including science, engineering, business, and aviation. Spreadsheets and many computer programs require formulas to analyze situations and predict patterns.

Answer:
Based solely on body-mass index, Helix's health is not at risk.

Figure This!

Get Started:

How many inches are in a foot? What is Helix's height in inches? In the formula, replace H with Helix's height in inches and W with his weight in pounds.

Complete Solution:

There are 12 inches per foot. Helix is 5 x 12 + 2, or 62 inches tall. He weighs 110 pounds. Using the formula, his body-mass index is approximately 20.

$$\text{BMI} = \frac{(110 \times 705) \div 62}{62} \approx 20$$

Because 20 is greater than 19 and less than 27, Helix's health is not at risk.

Try This:

- Find your own body-mass index. According to the result, should you be concerned about your health?
- Check a website for other indicators of health risk.

Additional Challenges:

1. Another formula for body-mass index is:

$$\text{BMI} = \frac{705W}{H^2}$$

Will Helix have a different body-mass index using this formula?

2. What is a healthy weight range for Helix's height?
3. According to body-mass index, for what heights would a weight of 180 pounds be considered safe?
4. How could you adjust the formula to use weight in kilograms and height in centimeters?

Things to Think About:

- What other factors are involved in determining health risk?
- Why does the formula for body-mass index involve dividing by height twice?

Did You Know That?

- According to *The Old Farmer's Almanac 2000*, a waist measurement of 35 inches or more in women and 41 inches or more in men, regardless of height, suggests a serious risk of weight-related health problems.
- Very muscular people often have a higher body-mass index because muscle weighs more than fat.
- One simple health-risk test is to pinch your side just above your waist. If you can pinch more than an inch, you are at risk of weight-related health problems.
- The amount of skin may be found using the formula $\sqrt{\dfrac{\text{height x weight}}{3600}}$ where height is in centimeters and weight is in kilograms. The result obtained is surface area measured in square meters. Weight is approximately proportional to height times height times height.
- Body-mass index is essentially a ratio of volume to surface area. The solid geometric figure with the greatest ratio of volume to surface area is the sphere.

Resources:

Books:

- "Media Clips." Edited by Dorothy Wood. *The Mathematics Teacher* 92 (March 1999): 234–235.
- Mathematical Sciences Education Board. *High School Mathematics at Work: Essay and Examples for the Education of All Students.* Washington, D. C.: National Academy of Sciences, 1998.
- *The Old Farmer's Almanac 2000.* Dublin, NH: Yankee Publishing, Inc., 1999.

Website:

- **www.sirius.on.ca/running/bmi_txt.html**

Answers to Additional Challenges:

(1.)
No. The formulas are equivalent.

(2.)
Using body-mass index, a safe range for weight is about 104 to 147 pounds.

(3.)
Between approximately 69 and 82 inches.

(4.)
To the nearest whole number, you can use 50 instead of 705 in the formula.

Math Challenge #22
COMBINATION LOCKS

FigureThis!
Math Challenges for Families

I forgot the combination! I'm ready to cry. How many combinations will I have to try?

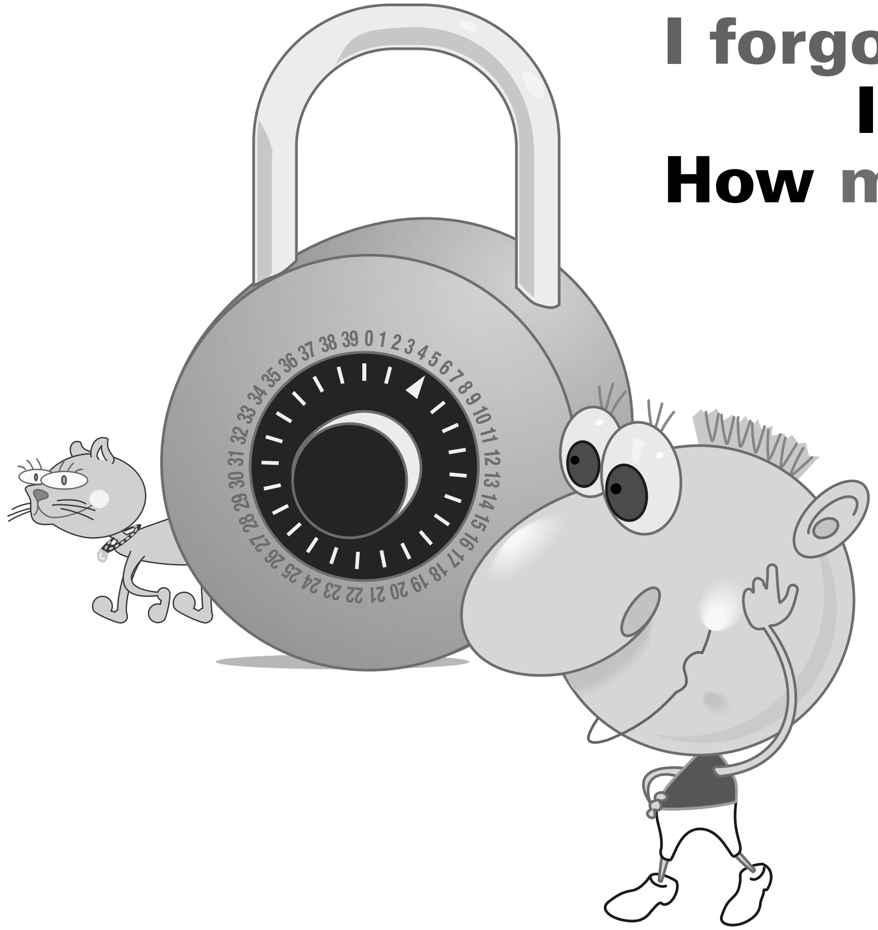


Figure This! This combination lock uses three numbers from 0 to 39. It opens when these numbers are dialed in a particular order: right, left, right. How many possible combinations are there?

Hint: Think about how many choices you have before dialing each number.

Counting the number of possible arrangements in a given situation is an important mathematical skill. Banks, phone companies, and security systems use codes based on such arrangements for Personal Identification Numbers (PIN).

Answer:
There are 64,000 different combinations for the lock.

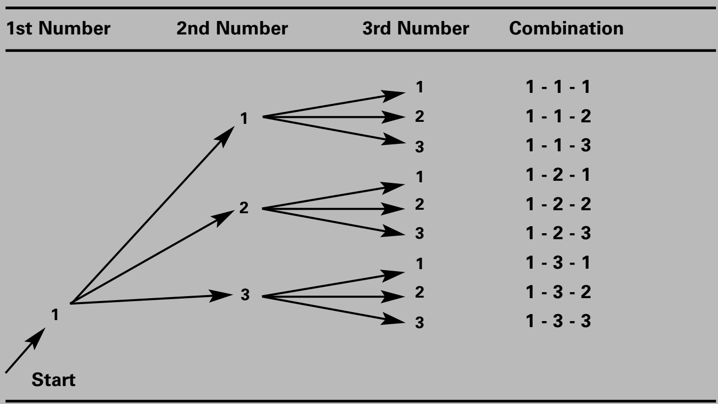
Figure This!

Get Started:

Try an easier problem. If the lock uses only the numbers 1, 2, and 3, how many different three-number combinations are possible? Write out the possible combinations in an ordered list and look for a pattern. What if the lock uses the numbers 1, 2, 3, and 4?

Complete Solution:

In the challenge, the lock uses the numbers 0 to 39. Start with an easier problem using 1, 2, and 3. You can count the possibilities by drawing a tree diagram. A portion of this tree diagram is shown below.



If you start with 1, you get nine different combinations. If you start with 2 instead of 1, you also get 9 different combinations. If you start with 3, you get 9 more possibilities for a total of 9 + 9 + 9, or 27 different combinations. Think about this in terms of choices. You can choose any of three numbers as a possible first number in the combination, follow that with a choice of any of the three numbers as the second number in the combination, and finally choose any of the three numbers for the final number in the combination for a total of 3 x 3 x 3, or 27 different combinations. If the lock used the numbers 1, 2, 3, and 4, you would have to choose from four numbers, three different times. This would give you 4 x 4 x 4, or 64 choices for the combination. The lock in the challenge requires that you choose from 40 different numbers, three different times. Therefore, there are 40 x 40 x 40, or 64,000 different combinations.

Try This:

• Experiment with a combination lock. Do you have to dial each number exactly?

Additional Challenges:

1. Suppose the combination for a particular brand of lock allows each number to appear only once. If the lock uses three numbers from 0 to 39 (as in the challenge), how many combinations are possible?
2. Suppose the combination for a bicycle lock is 10-24-32. With this lock, however, the numbers on either side work as well as the actual number. For example, the combinations 9-25-33 and 11-23-33 will also open the lock. How many different combinations will open this particular lock?
3. Some keypad locks have 10 keys numbered 0 to 9. You open the lock using 3 keystrokes. A keystroke can consist of pressing 1 key or of pressing 2 keys at the same time. How many combinations are possible on this lock?

Things to Think About:

- Why do you think that telephone companies have to add new area codes for certain regions?
- How many phone numbers do you think are served by each area code?
- Which kind of lock will allow more possible combinations: a keypad lock or a lock with a dial?
- Find all the locks in your house. How are they alike? How are they different?

Did You Know That?

- Mathematicians use the word *combination* differently than it is used in a "combination lock." In a mathematical combination, the order in which an item occurs is not important.
- Some old safes can be opened using more than one combination.
- A disc lock consists of a sequence of discs numbered on their outer edges. To open the lock, you turn the discs (usually three) to the appropriate numbers.
- Many apartment buildings, businesses, and airports—even restrooms—use some form of keypad lock.

Resources:

Websites:

- **www.howstuffworks.com/inside-lock.htm**
- **www.thelockman.com/cl1.htm**

Answers to Additional Challenges:

(1.)
40 x 39 x 38 = 59,280.

(2.)
$3^3 = 27$

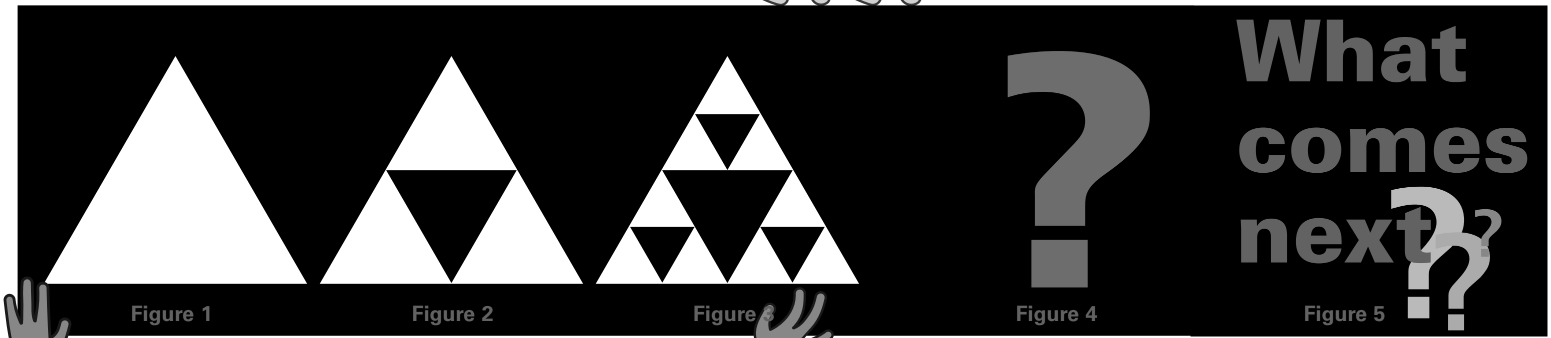
(3.)
$55^3 = 166,375.$

Math Challenge #23

FRACTALS

Figure This!

Math Challenges for Families



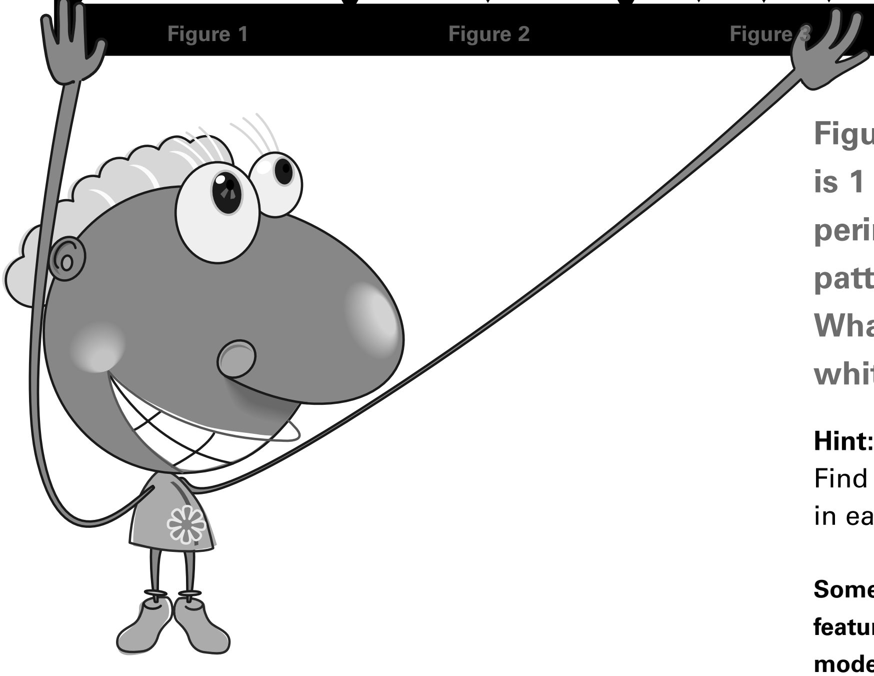


Figure This! If each side of the triangle in Figure 1 is 1 inch long, this means the triangle has a perimeter of 3 inches. Suppose you continued the pattern in the diagram until you reached Figure 5. What is the sum of the perimeters of all the white triangles in Figure 5?

Hint: The perimeter of a shape is the distance around it. Find the sum of the perimeters for all the white triangles in each of the figures above. What pattern do you see?

Some repeating patterns form fractals. Many naturally occurring features, such as ferns, weather patterns, or coastlines, can be modeled by fractals.

Answer:
15 3/16 inches

Figure This!

Get Started:

Figure 1 has only one white triangle. Since each side is 1 inch long, its perimeter is 3 inches. How many white triangles are in Figure 2? What is the perimeter of each of these triangles? What is the sum of the perimeters for Figure 2? Make a table to help answer the questions.

Figure Number	Side Length of Each White Triangle (inches)	Perimeter, *p*, of Each White Triangle = 3x Side Length (inches)	Total Number, *n*, of White Triangles	Sum, *np*, of the perimeters
1	1	3	1	3
2				

Complete Solution:

Complete the table in the hint.

Figure Number	Side Length of Each White Triangle (inches)	Perimeter, *p*, of Each White Triangle = 3x Side Length (inches)	Total Number, *n*, of White Triangles	Sum, *np*, of the perimeters
1	1	3	1	3
2	1/2	3/2	3	9/2
3	1/4	3/4	9	27/4
4	1/8	3/8	27	81/8
5	1/16	3/16	81	243/16

As the figure numbers increase, the side length of each white triangle is halved, and the number of white triangles is tripled. This means the sum of the perimeters in any particular figure is, 3 • 1/2 or 3/2 the sum in the previous figure. The sum of the perimeters in Figure 5 would be 243/16, or 15 3/16 inches.

Try This:

- As the pattern in the challenge continues, smaller copies of the first few figures repeat themselves. A similar effect occurs in some surprising places. For example, look at the front of a Cracker Jack™ box, or stand between two parallel mirrors and look at the images created. What patterns do you notice?
- Design your own fractal pattern.

Additional Challenges:

1. Suppose it takes 1 ounce of paint to cover the triangle in Figure 1, the original white triangle in the challenge. How much paint would you need to cover the white triangles in Figures 2 and 3?
2. Find a general rule for the sum of the perimeters of the white triangles in each figure in the challenge.
3. Each side of the square in Figure 1 below is 1 inch long. Suppose you continued the pattern in this diagram until you reached Figure 5.

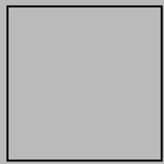


Figure 1

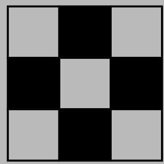


Figure 2

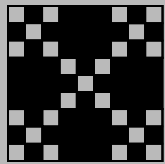


Figure 3

   a. Find the sum of the perimeters of the white squares in Figure 5.
   b. Find the sum of the areas of the white squares in Figure 5.

Things to Think About:

- If the pattern in the challenge continues, what do you think will eventually happen to the perimeter and area of successive figures?
- How do Bart Simpson's comic books provide an illustration of fractals? (See the references.)

Did You Know That?

- The pattern in the challenge results in a figure called Sierpinski's triangle, named after the Polish mathematician Waclaw Sierpinski, who developed it in about 1915.
- Parts of some fractals look like the whole fractal. This property is called self-similarity.
- Figures that can be built with smaller copies of the same figure are sometimes called rep-tiles or clones.
- Some computer illustrations use fractals to model clouds, coastlines and mountains.

Resources:

Books:

- Peitgen, H., H. Jurgens, D. Saupe, E. Maletsky, T. Perciante, and L. Yunker. *Fractals For The Classroom: Strategic Activities Volume One.* New York: Springer-Verlag, 1992.
- *Simpsons Comics Issue 4*. Los Angeles: Bongo Entertainment, Inc., 1994.
- *(Bart Simpson's) Treehouse of Horror Issue 2*. Los Angeles: Bongo Entertainment, Inc., 1996.

Website:

- **math.rice.edu/~lanius/fractals**

Answers to Additional Challenges:

(1.)
If it takes 1 ounce to paint the triangle in Figure 1, then it would take 3/4 of an ounce to paint the white triangles in Figure 2, and 9/16 of an ounce to paint the white triangles in Figure 3.

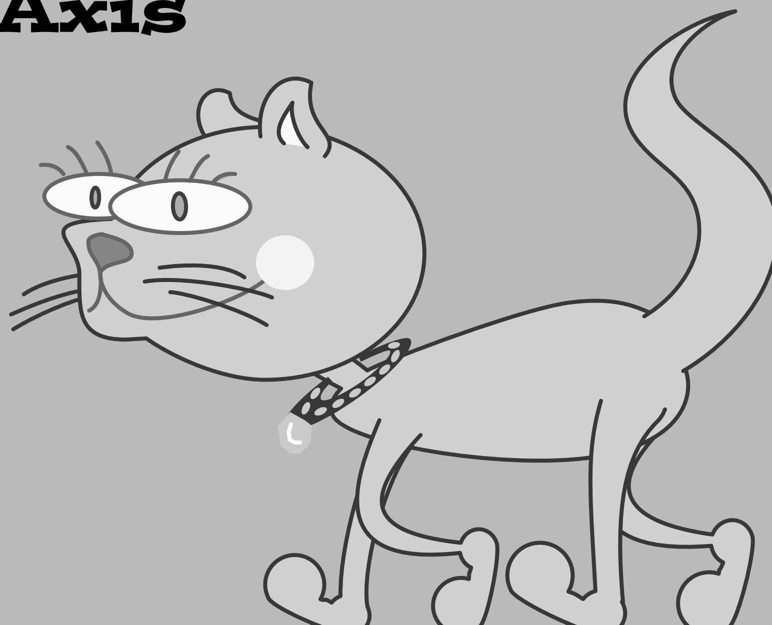
(2.)
If n is the figure number, and the side length in Figure 1 is 1 inch, then the sum of the perimeters, in inches, is: $\frac{3^n}{2^{n-1}}$ or $3 \cdot \left(\frac{3}{2}\right)^{n-1}$

(3 a.)
The sum of the perimeters for Figure 5 would be 2500/81 or about 30.9 inches.

(3 b.)
The sum of the areas is 625/6561, or about 0.1 square inches.

Notes:

Axis



Math Challenge #24 GASOLINE TANKS

Figure This! Math Challenges for Families

How far can you GO on a tank of gas?

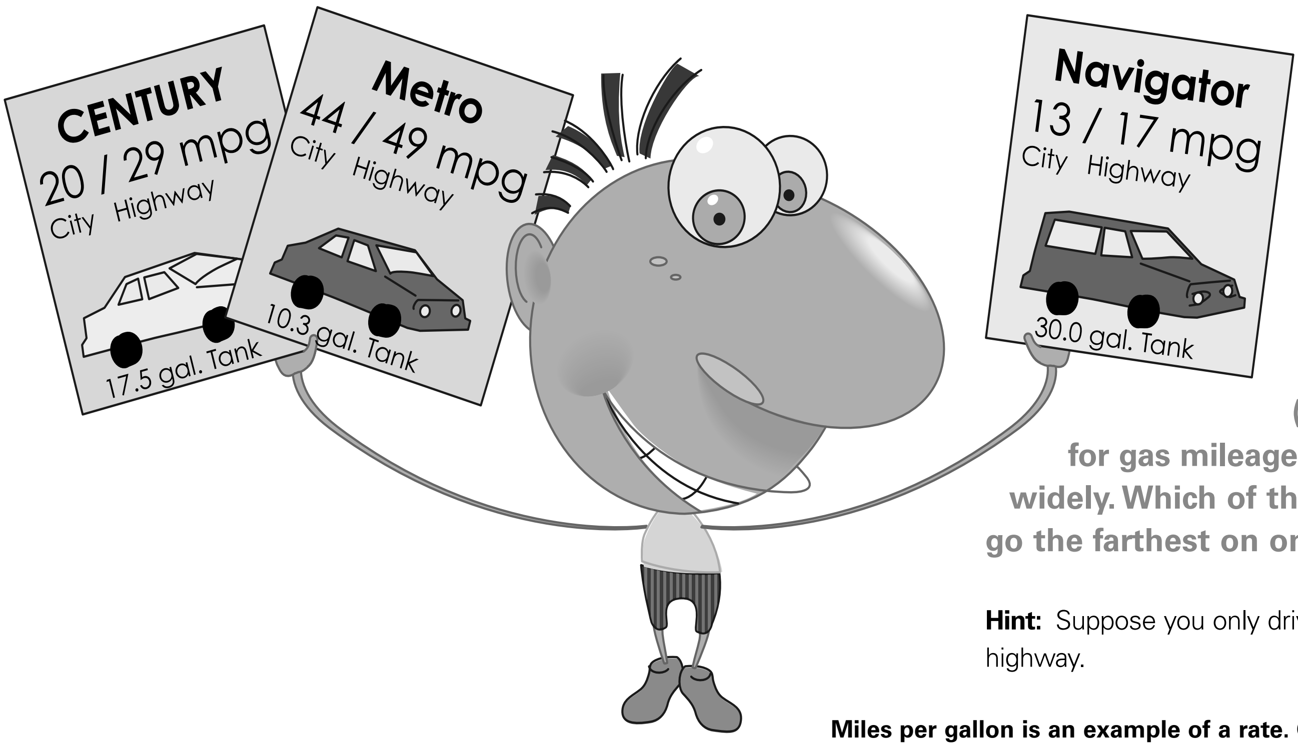


Figure This! The Environmental Protection Agency (EPA) estimates for gas mileage on 1999 cars vary widely. Which of these cars should go the farthest on one tank of gas?

Hint: Suppose you only drive in the city. On the highway.

Miles per gallon is an example of a rate. Grocers, demographers, financiers, actuaries, and economists all use rates in their work.

Answer:

For city driving, the Metro would go the farthest. For highway driving, the Navigator would go farthest although not significantly farther. You might also consider combinations of city and highway driving.

Figure This!

Get Started:

How many miles could each car travel on one tank of gas? You need to know its EPA mileage estimate, as well as its fuel capacity. The number of miles possible can be found using the formula: number of miles traveled = fuel capacity in gallons x the estimated mileage in miles per gallon.

Complete Solution:

Different combinations of city and highway driving are possible. If you assume all driving is done on the highway, the Navigator should go farthest as seen in the chart below, but all three go approximately the same distance.

CAR	Highway Mileage (mpg)	Tank Capacity (gallons)	Approximate Distance (miles)
Century	29	17.5	508
Metro	49	10.3	505
Navigator	17	30	510

If you assume all driving is done in the city, the Metro will go the farthest.

CAR	City Mileage (mpg)	Tank Capacity (gallons)	Approximate Distance (miles)
Century	20	17.5	350
Metro	44	10.3	453
Navigator	13	30	390

You might also consider different combinations of highway and city driving.

Try This:

- Choose a car. Use information from a website, magazine, or car dealer to determine how far this car can travel on one tank of gas.

Additional Challenges:

1. Suppose the following two cars travel 100 miles, half in the city and half on the highway. Which uses the least amount of gas?
   - Ferrari Maranello (9 mpg city / 14 mpg highway)
   - Lamborghini Diablo (10 mpg city / 13 mpg highway)
2. Suppose you wanted to design a car that could travel 600 highway miles on one tank of gas. What are some possible values for this car's fuel capacity and highway mileage?
3. Imagine that you drive a Metro. Your daily commute to work includes 80 highway miles and 10 city miles, each way. If you start the week with a full tank, on what day will you need to buy gas?

Things to Think About:

- How do automobile makers decide how many gallons of gas a tank should hold?
- How do automobile makers decide whether to put the gasoline tank on the left or the right of the car?
- Some experimental cars operate on electric batteries. Do these go farther before the batteries die than another car would go on a tank of gas?

Did You Know That?

- A 1991 Toyota Land Cruiser with a 38.2-gallon gas tank traveled 1691.6 miles without refueling.
- Volkswagen has developed a car that can travel 80 miles on 1 gallon of gas.
- Many trucks have more than one gas tank.
- Because they had no gas gauge, some older-model Volkswagens had a 2-gallon reserve tank.

Resources:

Books:

- *The Guinness Book of Records*. New York: Guinness Book of Records, 1999.
- *Kelley Blue Book Used Car Guide January-June 2000: Consumer Edition*.
- *N.A.D.A. Official Used Car Guide*. McLean, VA: National Automobile Dealers Association.
- *Consumer Reports*. Yonkers, NY: Consumers Union.

Websites:

- **www.edmunds.com**
- **www.consumerreports.org**
- **www.epa.gov/OMSWWW/mpg.htm**
- **www.newcarpoint.com/index.html**

Answers to Additional Challenges:

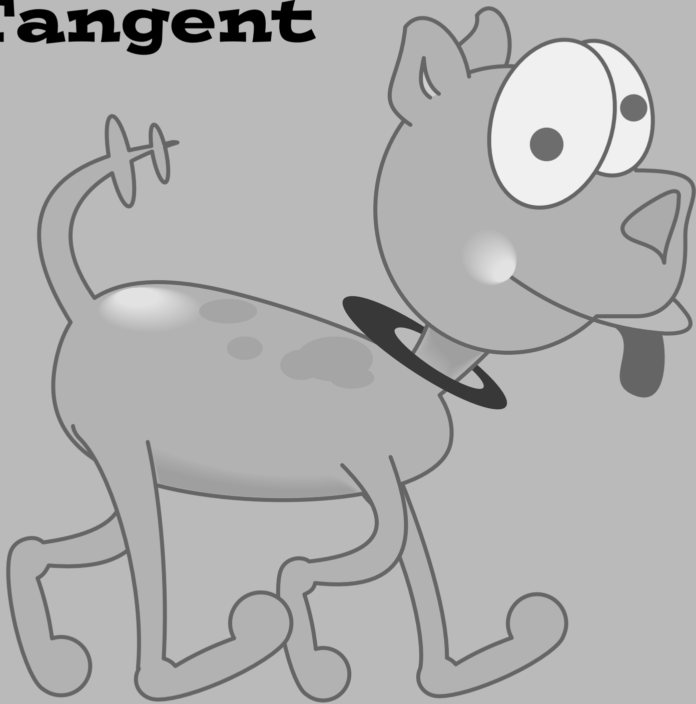
(1.)
The Lamborghini Diablo.

(2.)
There are many combinations possible. For example, a car with a 13.4 – gallon tank and highway mileage of 45 mpg could go about 600 miles.

(3.)
You will need gas on the third day.

Notes:

Tangent



Math Challenge

#25

GRAPE JUICE JUNGLE

Figure This!
Math Challenges for Families

Which tastes JUICIER?

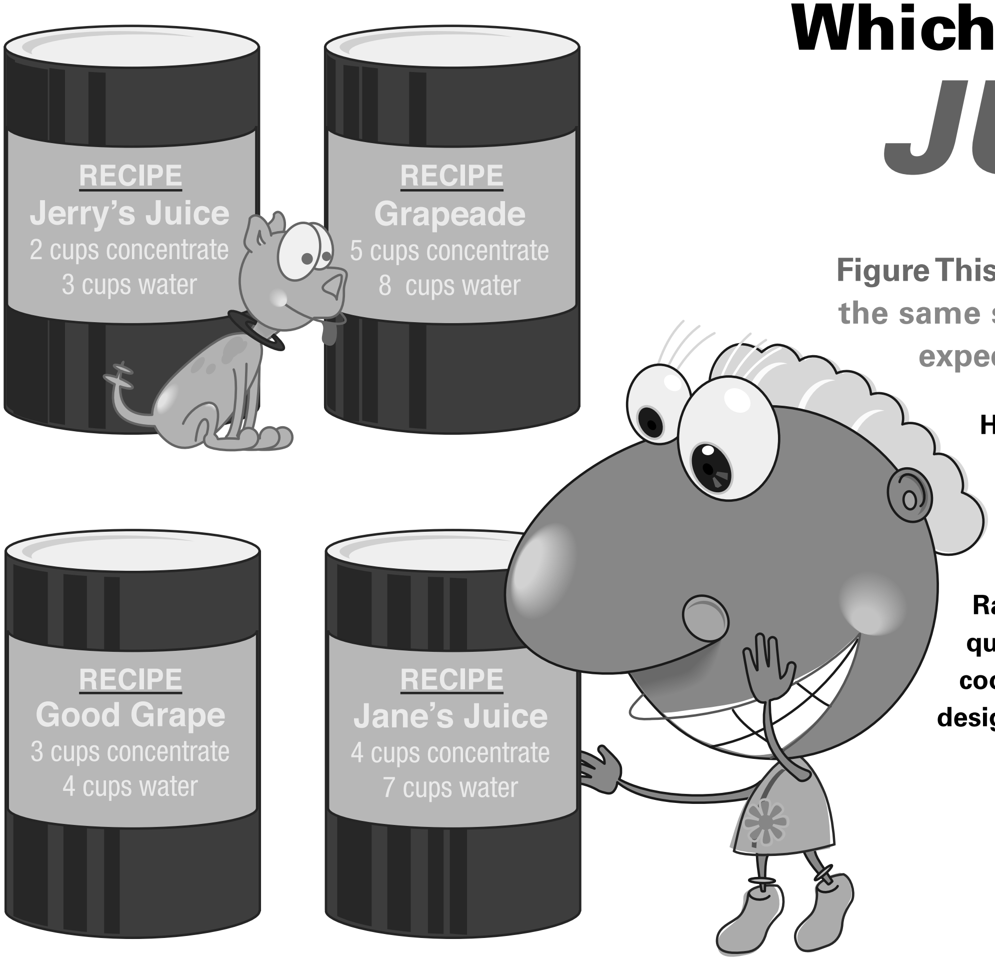


Figure This! If all grape juice concentrates are the same strength, which recipe would you expect to have the strongest grape taste?

Hint: For each recipe think about how much water should be used with 1 cup (c.) of concentrate, or how much concentrate should be used with 1 cup of water.

Ratios are fractions that compare two or more quantities. Shoppers use ratios to compare prices; cooks use them to adjust recipes. Architects and designers use ratios to create scale drawings.

Answer:
Good Grape should have the strongest grape taste.

Figure This!

Get Started:

Answer one of the following questions: Which recipe uses the most water for 1 cup of concentrate? Which recipe uses the most concentrate for 1 cup of water?

Complete Solution:

There are several ways to approach this problem.

- One way is to determine how much concentrate each recipe uses for 1 cup of water. The one that uses the most concentrate should have the strongest grape taste.

RECIPE	Cups of Concentrate per Recipe	Cups of Water per Recipe	Ratio of Concentrate to Water	Ratio of Concentrate to 1 cup of Water
Jerry's Juice	2	3	2/3	$\frac{2 \div 3}{3 \div 3} \approx \frac{0.67}{1}$
Grapeade	5	8	5/8	$\frac{5 \div 8}{8 \div 8} \approx \frac{0.63}{1}$
Good Grape	3	4	3/4	$\frac{3 \div 4}{4 \div 4} = \frac{0.75}{1}$
Jane's Juice	4	7	4/7	$\frac{4 \div 7}{7 \div 7} \approx \frac{0.56}{1}$

Good Grape has the most concentrate (0.75) for 1 cup of water. It should have the strongest grape taste.

- Another way is to find how much water each recipe uses for 1 cup of concentrate. Here, the recipe that uses the least water should have the strongest grape taste.

RECIPE	Cups of Concentrate per Recipe	Cups of Water per Recipe	Ratio of Concentrate to Water	Ratio of 1 Cup of Concentrate to Water
Jerry's Juice	2	3	2/3	$\frac{2 \div 2}{3 \div 2} = \frac{1}{1.5}$
Grapeade	5	8	5/8	$\frac{5 \div 5}{8 \div 5} = \frac{1}{1.6}$
Good Grape	3	4	3/4	$\frac{3 \div 3}{4 \div 3} \approx \frac{1}{1.3}$
Jane's Juice	4	7	4/7	$\frac{4 \div 4}{7 \div 4} \approx \frac{1}{1.8}$

Good Grape has the least amount of water, 1.3 cups, to 1 cup of concentrate and so should have the most grape flavor.

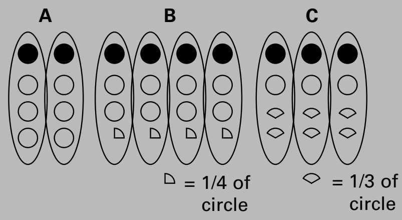
Try This:

- Choose a product such as breakfast cereal, liquid soap, or canned soup. Check several different brands and package sizes to see the differences in the cost per unit of weight or the cost per unit of volume.
- A box of oatmeal usually has recipes for different sized servings. Check a box to see if the proportions of ingredients are the same for the different sized servings.

Additional Challenge:

1. What is the ratio of water to the total amount of liquid in one recipe of Jerry's Juice?

2. A fifth grade student drew this picture to solve a problem like the one in the challenge. The dark circles represent concentrate, and the white circles represent water.



Describe each recipe. Rank the recipes in order from strongest to weakest.

3. If we mixed the concentrates for Jerry's Juice and Grapeade and made a recipe, is this mixture as strongly grape flavored as Jerry's Juice alone?

Things to Think About:

- What fraction of the human body is water?
- What proportion of fats, carbohydrates, and proteins do you eat in a normal meal?
- Do all lemonade mixes contain lemon?

Did You Know That?

- Grape juice and apple juice provide the base for many flavors of fruit juices.
- If a recipe is doubled, tripled, or quadrupled, the proportion of an ingredient stays the same. This is an example of a direct proportion.
- Direct proportions lead to the study of lines or linearity.

Resources:

Books:

- Keijzer, Ronald, Mieke Abels, Laura J. Brinker, S. R. Cole, and Julia A. Shew. *Ratios and Rates.* In *Mathematics in Context.* National Center for Research in Mathematical Sciences Education and Freudenthal Institute. Chicago: Encyclopaedia Britannica Educational Corporation, 1998.
- Lappan, G., J. Fey, W. Fitzgerald, S. Friel, and R. Phillips. *Connected Mathematics: Comparing and Scaling.* Palo Alto, CA: Dale Seymour Publications, 1996.
- Niven, Ivan. *Numbers: Rational and Irrational.* New Mathematical Library, Vol. 1. Washington, DC: Mathematical Association of America, 1961.

Answers to Additional Challenges:

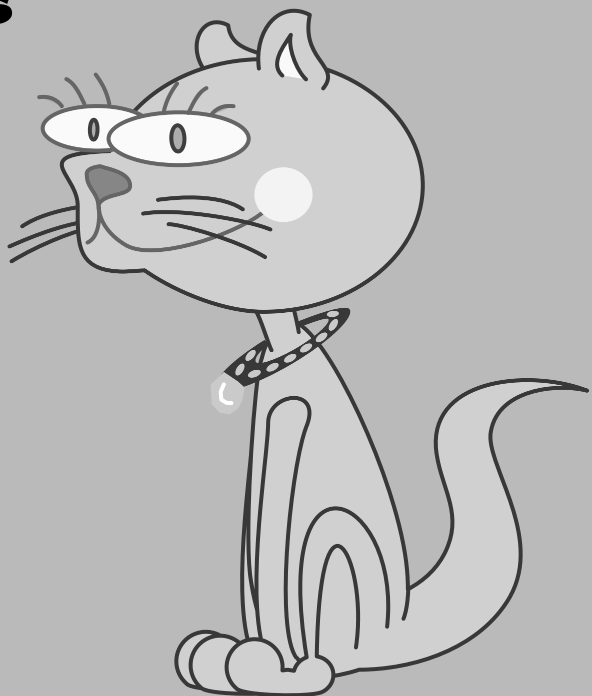
(1.)
The ratio is 3 to 5.

(2.)
A is 2 cups of concentrate to 6 cups of water. B is 4 cups of concentrate to 9 cups of water. C is 3 cups of concentrate to 5 cups of water. The ranking is C, B, and A from strongest to weakest.

(3.)
Jerry's Juice is stronger than the mixture.

Notes:

Axis



Math Challenge #26 I WIN!

FigureThis! Math Challenges for Families

"SHE always wins. It's not fair!"

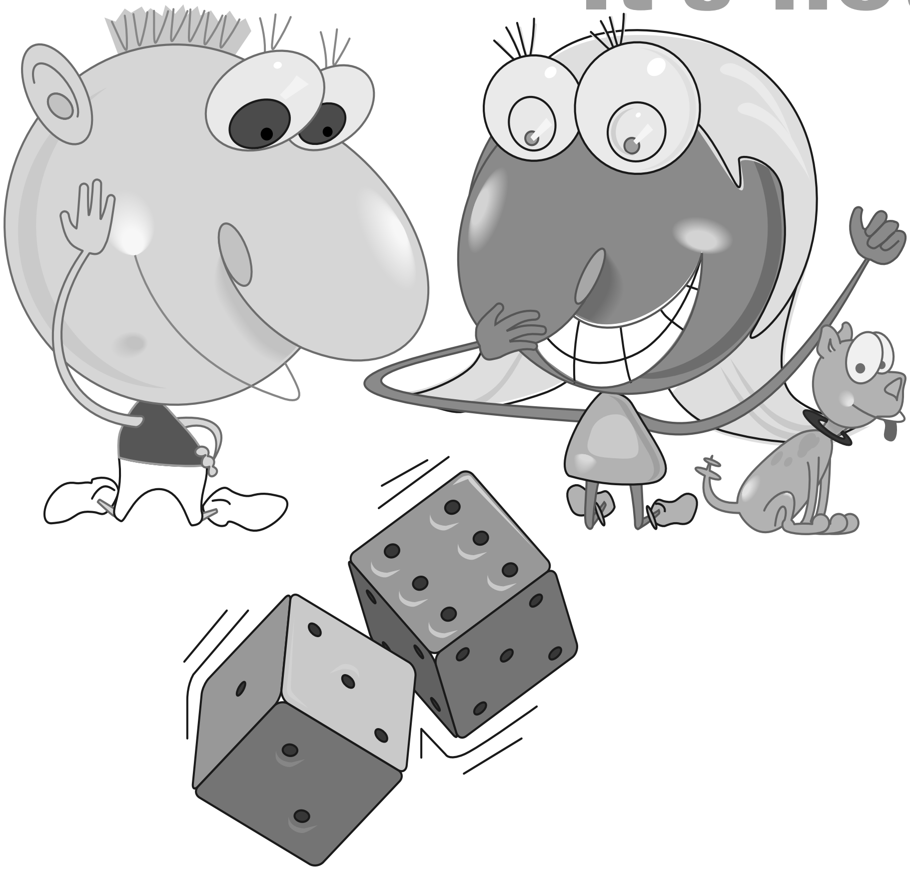


Figure This! Two players each roll an ordinary six-sided die. Of the two numbers showing, the smaller is subtracted from the larger. If the difference is 0, 1, or 2, player A gets 1 point. If the difference is 3, 4, or 5, Player B gets 1 point. The game ends after 12 rounds. The player with the most points wins the game. Is this game fair?

Hint: In a fair game, all players are equally likely to win. Play this game several times and record the results of each roll.

Math can help determine whether or not a game is fair. Math can also determine fairness in other situations, such as assigning seats in the U.S. House of Representatives or settling an estate.

Answer:
This game is not fair. Player A is twice as likely as player B to win the point for each round.

Figure This!

Get Started:

If player A rolls a 1 and player B rolls a 6, the difference is 5. Make a table that shows all the possible differences when rolling two dice.

DIFFERENCE TABLE

PLAYER A \ PLAYER B	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Use the table to determine if one player is more likely to win.

Complete Solution:

Create a table that shows all the possible differences that can result from the roll of two dice.

DIFFERENCE TABLE

PLAYER A \ PLAYER B	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

Of the 36 possible outcomes, the differences 0, 1, and 2 appear 24 times, while the differences 3, 4, and 5 appear only 12 times. Player A is twice as likely to win a point on each roll and is therefore much more likely to win the game.

Try This:

- Change the rules of the game in the Challenge. Play it again. Do the new rules make a difference in who wins?

Additional Challenges:

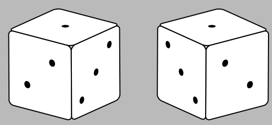
1. Change the rules of the game in the challenge so that the game is fair.
2. Roll two standard six-sided dice and divide the larger number showing by the smaller number. Player A gets 1 point if there is a remainder other than 0. Player B gets 1 point if the remainder is 0. Is this a fair game? If the game is not fair, how would you make it fair?
3. Assign numbers to the faces of two dice so that, on any one roll if you add the numbers on the top faces, each sum from 1 to 12 is equally likely.

Things to Think About:

- Is the game of Monopoly™ fair?
- There is no winning strategy that guarantees a win for the game Rock-Paper-Scissors. Why do you think this is true?
- Could you create a die with an odd number of faces?

Did You Know That?

- The opposite faces of an ordinary six-sided die always add up to 7.
- There are "right-handed" and "left-handed" dice. In Europe and the Americas, dice are typically right-handed. This means that if the 1 is face up, and the 2 is facing you, then the 3 will be on your right.



Right Hand Die Left Hand Die

In Asia, dice typically have the 3 on the left.

- A branch of mathematics known as probability got its start in 1654, when Blaise Pascal and his friend, the Chevalier de Méré, tried to analyze a game of dice.
- Dice were developed from ancient games played with sheep knucklebones, called *astragali* in Greek, and *tali* in Latin. The bones from sheep's ankles were used to tell fortunes. Called "rolling bones," they were basically rectangular and had numbers on only four sides.
- A die with four faces is a tetrahedron. A die with twelve faces is an dodecahedron. A die with twenty faces is an icosahedron.

Resources:

Books:

- Mohr, Merilyn Simonds. *The Games Treasury*. Shelburne, VT: Chapters Publishing, Ltd., 1993.
- Crisler, Nancy, Patience Fisher, and Gary Froelich. *Discrete Mathematics Through Applications*. New York: W. H. Freeman and Co., 1994.

Website:

- **www.gamecabinet.com/rules/Dice.html**

Answers to Additional Challenges:

(1.)
There are many ways to make this game fair. For example, you might give 1 point to player A for an even difference and 1 point to player B for an odd difference. Or you might give 1 point to player A for differences of 0, 3, 4, or 5, and 1 point to player B for differences of 1 or 2.

(2.)
The game is not fair. Of the 36 possible outcomes, the remainder 0 appears 22 times, the remainder 1 appears 10 times, and the remainder 2 appears 4 times. One way to make this game fair would be to give player A 7 points for a remainder of 0 and player B 11 points for a remainder of 1 or 2.

(3.)
There are several ways to do this. For example, you could label one die with the numbers from 1 to 6 and the other with three 0s and three 6s.

Notes:

Tangent

Math Challenge #27

LIFE EXPECTANCY

Figure This! Math Challenges for Families

Do women live longer than men?

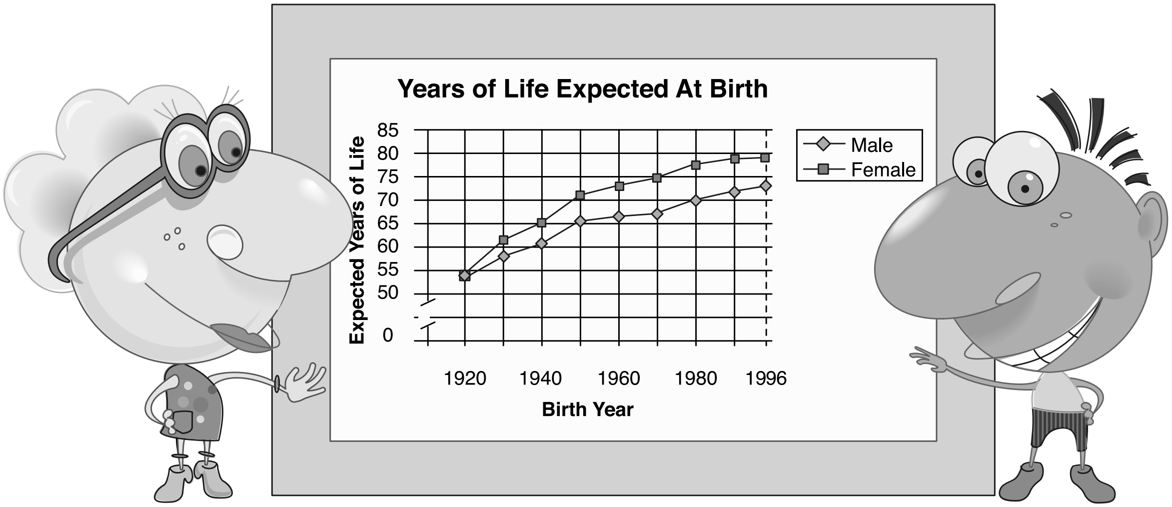


Figure This! This chart shows the life expectancy for persons born in the United States in a given year. Estimate the biggest difference between the life expectancy of men and women in any year from 1920 to 1996.

Hint: A woman born in 1920 had a life expectancy of about 55 years at birth.

Graphs of information over time are useful in identifying patterns and analyzing trends. Market research firms, radio stations, and breakfast-food manufacturers all keep track of trends in order to stay current.

Answer:
In both 1970 and 1980, the difference in life expectancy was between 7 and 8 years.

Figure This!

Get Started:

Think about what the graph means.

- What was the life expectancy for males born in 1940? for females? What was the difference?
- What would it mean if the lines representing males and females crossed?

Complete Solution:

Look at each pair of data points and find the difference in life expectancy for males and females. You can estimate the difference from the graph or measure it with a ruler. You can also use an index card or a sheet of paper and mark the differences to see which is greater. Then identify the years for which the greatest difference occurs. The greatest difference (between 7 and 8 years) appears in both 1970 and 1980.

Try This:

- Look in newspapers and magazines to find a graph describing how something changes over time. What can you observe from the graph?

Additional Challenges:

1. Would the answer in the challenge be different if the scale on the graph were changed?
2. Describe what a graph of male and female life expectancies would look like if the difference were always six years.

Things to think about:

- Do you think the life expectancies of both men and women will continue to increase in the future?
- Why do women tend to live longer than men?
- What factors may have caused life expectancy to increase since 1920?
- How do you think life expectancy is determined?
- What would you expect a graph of life expectancies for another country to look like?

Did You Know That?

- Franklin Delano Roosevelt signed the Social Security Act into law in 1935.
- As of 1999, a woman named Jeanne Calment held the record for longest life. She lived 122 years and 164 days. She died August 4, 1997.

Resources:

Books:

- *The Guinness Book of World Records*. New York: Bantam Books, 1998.
- *The World Almanac and Book of Facts 1999*. Mahwah, NJ: World Almanac Books, 1998.

Website:

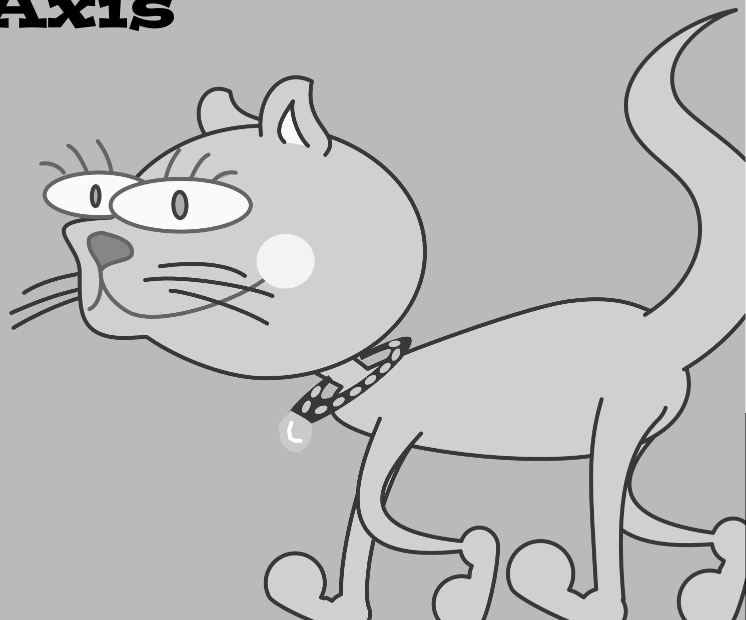
- **www.psych.cornell.edu/Darlington/lifespan.htm**

Answers to Additional Challenges:

(1.)
No. The result would be the same.

(2.)
The graph for men and the graph for women would have the same shape, with one 6 units above the other.

Axis



Math Challenge
#28
PIGEONHOLES

Figure This!
Math Challenges for Families

At least two people in a school have the same initials? No way!

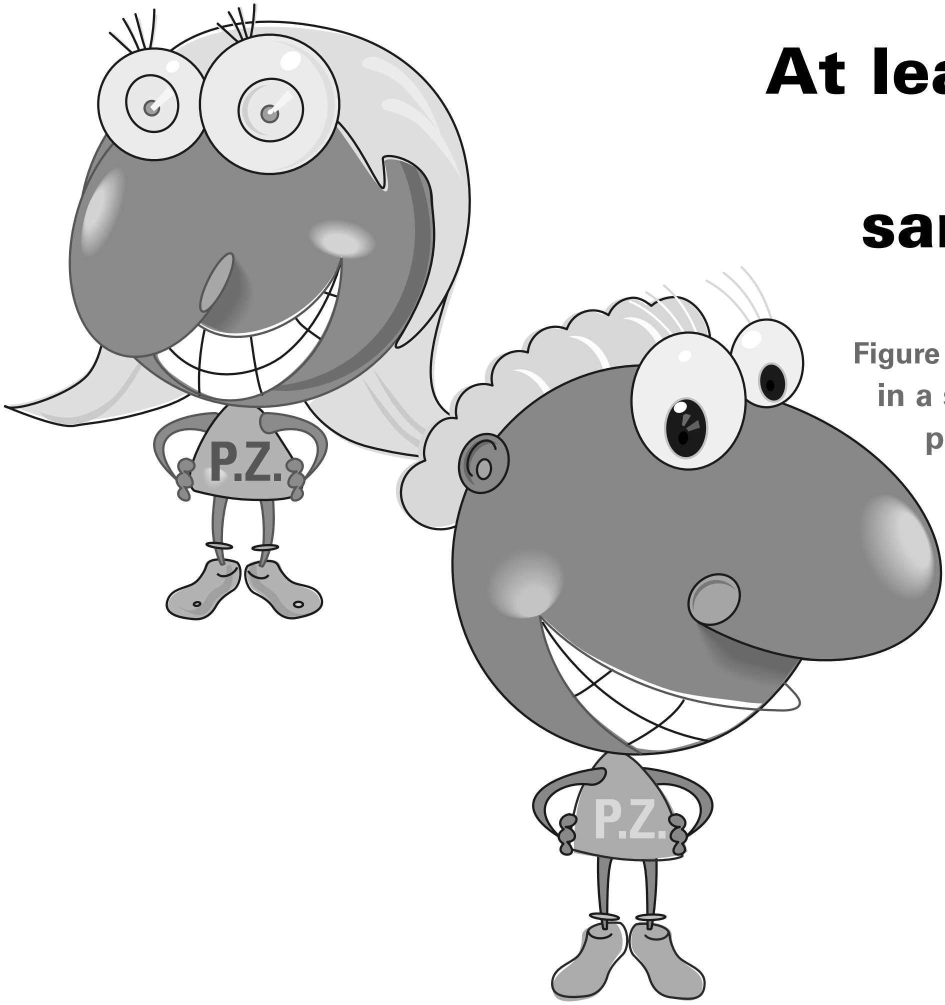


Figure This! How many people would have to be in a school before it contained at least two people with the same first and last initials?

Hint: Consider a simpler problem. How many people would have to enter a room before it contained at least two people with the same first initial?

The "pigeonhole principle" says that if you are putting objects into boxes and you have more objects than boxes, then at least one box will contain more than one object. People apply this principle when they mail announcements, pack shipping crates, or organize files.

Answer:
At least 677 people must be in a school.

Figure This!

Get Started:

How many different possibilities are there for a first initial? for a last initial? for both initials combined?

Complete Solution:

- There are 26 letters in the English alphabet. So there are 26 different possibilities for the first initial. Consider all the possible pairs of two initials. For example, suppose a person has the first initial A. Then the pair of initials could be AA, AB, AC, …, AZ. There are 26 different possibilities. If the first initial is B, the pair of initials could be BA, BB, BC, …, BZ. Again there are 26 different pairs. Continuing in this way and since there are 26 possible first initials, each of which could be paired with 26 last initials, there are 26 x 26, or 676 possible different pairs of initials. If there were 677 people, at least two of them must have the same pair of initials.

- Another way to begin this problem is to think about a situation involving smaller numbers. Suppose you have 11 items (that cannot be broken) to give to 10 people. This means that 1 person will get 2 of the items. The same reasoning can be used to solve the challenge.

Try This:

- Search the web to determine the estimated number of telephones in your state. If you know the number of households in your state, what conclusions can you reach about the number of telephones per household?
- Search the web or look in an encyclopedia to find the average number of hairs on a human head. Considering this information, do you think there are at least two people with the same number of hairs on their heads in your town? What about when your local stadium is filled?
- Search for "pigeonhole principle" on the web.

Additional Challenges:

1. How many people must enter a room to guarantee that at least two of them have the same birthday, regardless of whether or not they were born in a leap year?
2. A drawer contains 11 black socks and 3 gray socks. How many socks must you take out to guarantee that you have at least one pair of the same color?
3. A basketball team has 12 players. The team's jerseys are numbered from 0 to 20. If no two players are assigned the same number, must two players have consecutive numbers?

Things to Think About:

- Why do some states use both letters and numbers on their automotive license plates?
- At any party consisting of 2 or more people, at least 2 of the people must have the same number of friends at the party.
- Given any 17 whole numbers, you can find five of them whose sum is a multiple of 5.

Did You Know That?

- French mathematician Peter Gustav Lejeune Dirichlet (1805–1859) first described the pigeonhole principle.
- The pigeonhole principle was so named because if 10 homing pigeons return to 9 holes, then at least one hole must have two pigeons in it.
- A good approximation of the number of hairs on a human head is about 100,000.

Resources:

Books:

- Engel, Arthur. "The Box Principle." In *Problem-Solving Strategies*. New York: Springer-Verlag, 1991. pp. 39-58.
- Gardner, Martin. *Aha! Insight*. New York: Scientific American, Inc., 1978.
- Larson, Loren C. "Pigeonhole Principle." In *Problem-Solving Through Problems*. New York: Springer-Verlag, 1981. pp. 79-83.

Website:

- **www.optonline.com/comptons/ceo/02055_A.html**

Answers to Additional Challenges:

(1.) 367

(2.) 3

(3.) Yes

Math Challenge #29 SALARIES

Figure This! Math Challenges for Families

Will women ever earn as much money as men?

Figure This! A newspaper headline reads:"Women's salaries catching up to men's." Using the information below from the U. S. census, do you think this is true?

Median Salaries for Workers 25 or Over According to Both Year and Gender:

YEAR	MEDIAN SALARY: WOMEN	MEDIAN SALARY: MEN
1991	$11,580	$23,686
1992	$11,922	$23,894
1993	$12,234	$24,605
1994	$12,766	$25,465
1995	$13,821	$26,346
1996	$14,682	$27,248
1997	$15,573	$28,919

Hint: Think of ways to describe and compare the growth in salaries.

Organizing and interpreting information is a critical skill for business, industry, and many other professions. Making claims or decisions based on data is done by advertisers, insurance companies, athletic teams, and manufacturers.

Answer:
Based only on the data given, the statement appears to be untrue, at least in the near future. Over the long run, however, it appears that women's salaries may reach or exceed men's if the trends indicated by these data continue.

Figure This!

Get Started:

Think about either differences or ratios. What are the differences in men's and women's salaries from year to year? Compare women's salaries to men's salaries by making a fraction. What are the differences between men's and women's salaries over the entire time from 1991 to 1997? How do the fractions change? Think about percent increase as well as the difference in dollars.

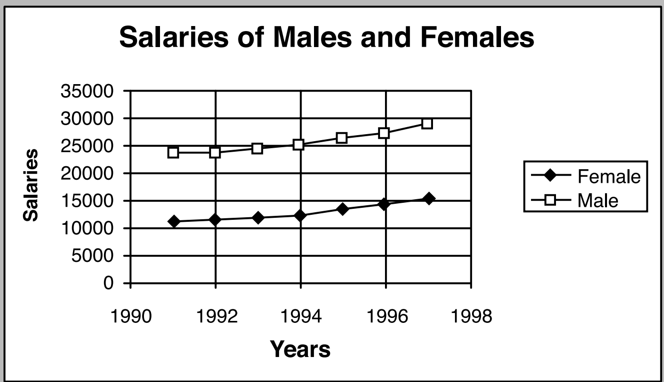
Complete Solution:

There are several ways to think about this problem. Your answer depends on your assumptions.

- Find the difference between men's and women's salaries each year; then look for a trend. Using this method, the gap appears to be growing in favor of men.

YEAR	MEDIAN SALARY: WOMEN	MEDIAN SALARY: MEN	MEN'S SALARIES – WOMEN'S SALARIES
1991	$11,580	$23,686	$12,106
1992	$11,922	$23,894	$11,972
1993	$12,234	$24,605	$12,525
1994	$12,766	$25,465	$12,371
1995	$13,821	$26,346	$12,699
1996	$14,682	$27,248	$12,566
1997	$15,573	$28,919	$13,346

- Another way to think about the data is to draw a graph with the year on the horizontal axis and salaries on the vertical axis. Plot the salary data for women, then connect the data points in order. On the same graph, plot and connect the data points for men's salary. As shown in the following graph, men's salaries are greater than women's for every data point. From the data up to 1998, the two graphs look like they will never cross, which may lead you to predict that men's salaries will remain higher than women's. You may need more information to predict the future.



- A third way to analyze this information is to compare the percent increase in salaries for both men and women. For the period from 1991 to 1997, the percent increases are:

$$\text{Men: } \frac{28{,}919 - 23{,}686}{23{,}686} \approx 22\%$$

$$\text{Women: } \frac{15{,}573 - 11{,}580}{11{,}580} \approx 34.5\%$$

If this trend continues, women's salaries will eventually meet and then exceed men's salaries.

Try This:

- Find information on salaries for a particular career. (Check with your school guidance department, the Internet, or an almanac.) Is there a difference in the salaries for men and women?

Additional Challenges:

1. Using the data in the challenge, find the percent increase in salary for men and for women in every year. What is the average annual percent increase in salary for men and women?
2. The table below shows predicted salaries for men and women using the average annual percent increases from 1991 to 1997. What do these predictions indicate about men's and women's salaries in the future?

YEAR	WOMEN'S PROJECTED SALARIES	MEN'S PROJECTED: SALARIES
2010	$29.365	$44,663
2020	$47,832	$62,395
2030	$77,913	$87,167

Things to Think About:

- What affects the trends in salaries?
- What might cause women's salaries to be lower than men's?
- How do the salaries of male and female athletes compare?
- Are there many women in the list of the world's richest people?
- Why do you think salaries for men and women are usually represented by the median (or middle value), rather than the mean (or average)?

- A 1% salary increase is not necessarily less than a 50% salary increase—it depends on the original salaries involved. (For example, compare 1% of $1,000,000 to 50% of $10,000.)

Did You Know That?

- In 1996, the maximum German hourly pay rate of $31.87 for those in manufacturing was the highest in the world. The corresponding rate in the United States was $17.70.
- In 1996, residents of New York City had the highest average annual salary in the United States: $45,028. Among urban areas, residents of Jacksonville, North Carolina, had the lowest average annual salary: $17,934.
- Mathematicians sometimes use an exponential model to describe rates of growth over time. In some cases, in which the increases level off, a logistic model is used.

Resources:

Books:

- T*he Guinness Book of World Records*. New York: Bantam Books, 1998.
- *The World Almanac and Book of Facts, 1999*. Mahwah, NJ: World Almanac Books, 1998.

Website:

- **www.census.gov/income/p13.txt**

Answers to Additional Challenges:

(1.)

For men, the average annual increase is about 3.4%; for women, the average increase is about 5%.

(2.)

If salaries continue to grow at past rates, women's salaries will be equal to men's in about 2036.

Notes:

Math Challenge #30 SMILES

Figure This! Math Challenges for Families

Which is worth more, a SMILE or a FROWN?

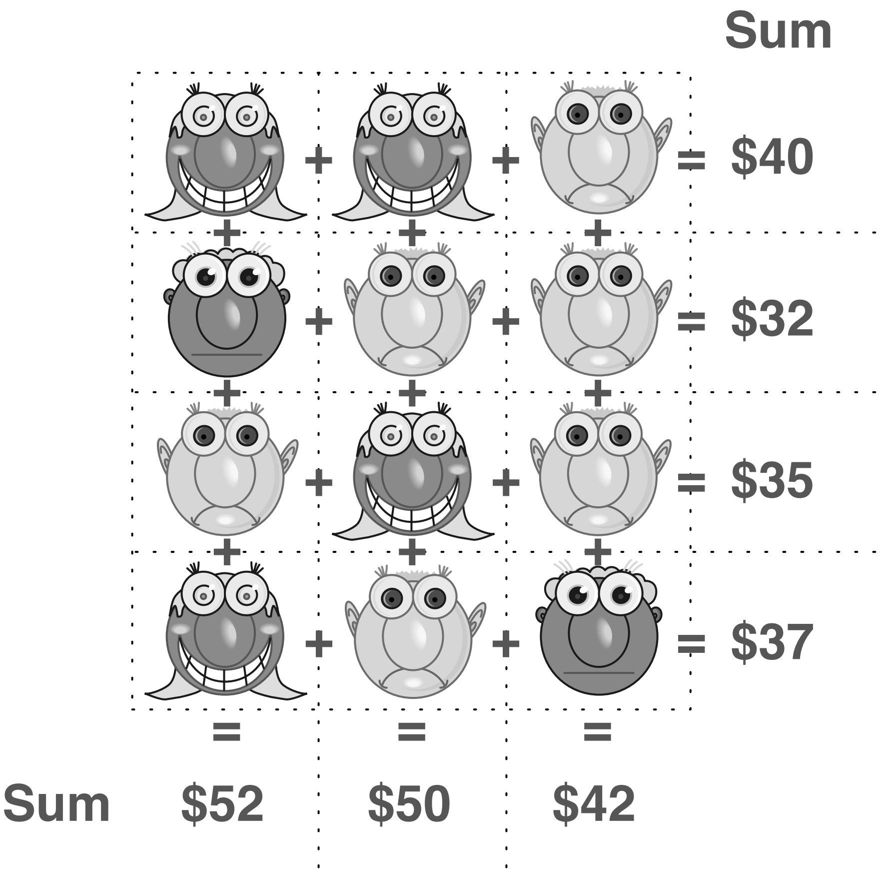


Figure This! The costs of combinations of frowns, smiles, and neutral faces are shown. How much is a smile worth?

Hint: Find a way to combine two of the rows or columns that have something in common.

Reasoning about unknowns is essential in studying equations. Economists, nurses, chemists, and engineers all use equations in their work.

Answer:
A smile is worth $15. A frown is worth $10.

Figure This!

Get Started:

- Look at the rows and columns. Which rows or columns have only smiles or frowns? What is the same and what is different in these rows and columns? Can you see a pattern?

Complete Solution:

There are many ways to approach this challenge.

- Top row ➞ 2 smiles and 1 frown = $40.
  Middle column ➞ 2 smiles and 2 frowns = $50.
  The middle column has one more frown and costs $10 more than the top row, so:
  1 frown = $10.
  Since 2 smiles and 1 frown = $40
  and 1 frown = $10,
  Subtracting 2 smiles = $30, so that
  1 smile = $15.

- Another way is to use the first and third rows.
  2 smiles and 1 frown = $40
  1 smile and 2 frowns = $35
  Continue the pattern, lose a smile, and add a frown for $5 less.
  0 smiles and 3 frowns = $30
  1 frown = $10
  Then continue using the the first method to find the cost of a smile.

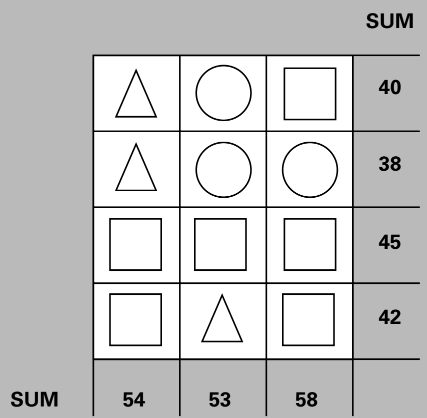
Try This:

- Choose values for frowns, smiles, and neutral faces, then make a grid of your own. See if a friend can solve it.

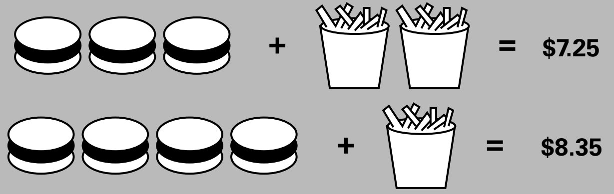
Additional Challenges:

1. Find the value of a neutral face in the challenge.

2. Find the value of each shape in the following diagram.



3. The pictures below show the cost of hamburgers and bags of fries. How much does a hamburger, and how much does a bag of fries cost?



Things to Think About:

- Unknowns in math can be represented by pictures, words, or letters.
- Many people use algebra in daily work without thinking about it.
- Are there other combinations in the original challenge that would lead to the solution?
- Does rearranging the rows in the Challenge make a difference? The columns?

Did You Know That?

- Many books and magazines feature puzzles like the one in the challenge.
- Some people earn their livings creating puzzles and logic problems.
- The mathematical study of linear programming, which involves systems of equations, is used by business and industry for decision making.
- Game theory and logic are branches of math where games and puzzles are found.

Resources:

Book:

- Skitt, Gale, Harold Skitt, and Carolyn Skitt. *Mensa Math Games for Kids*. Rocklin, CA: Prima Publishing, 1994.
- Salny, Abbie. *Cranium Crackers*. New York: Dodd, Mead and Co., 1986.
- Sawyer, W.W. *Vision in Elementary Mathematics*. Baltimore, MD: Penguin Books, 1964.

Magazine:

- *Games*. Ambler, PA: Games Publications, Inc.

Answers to Additional Challenges:

(1.) :
$12

(2.) :
Each square is worth 15, each triangle is worth 12, and each circle is worth 13.

(3.) :
One hamburger costs $1.89 and one bag of fries costs $0.79.

Math Challenge #31 TOURNAMENTS

FigureThis!
Math Challenges for Families

I've got to win how many games?

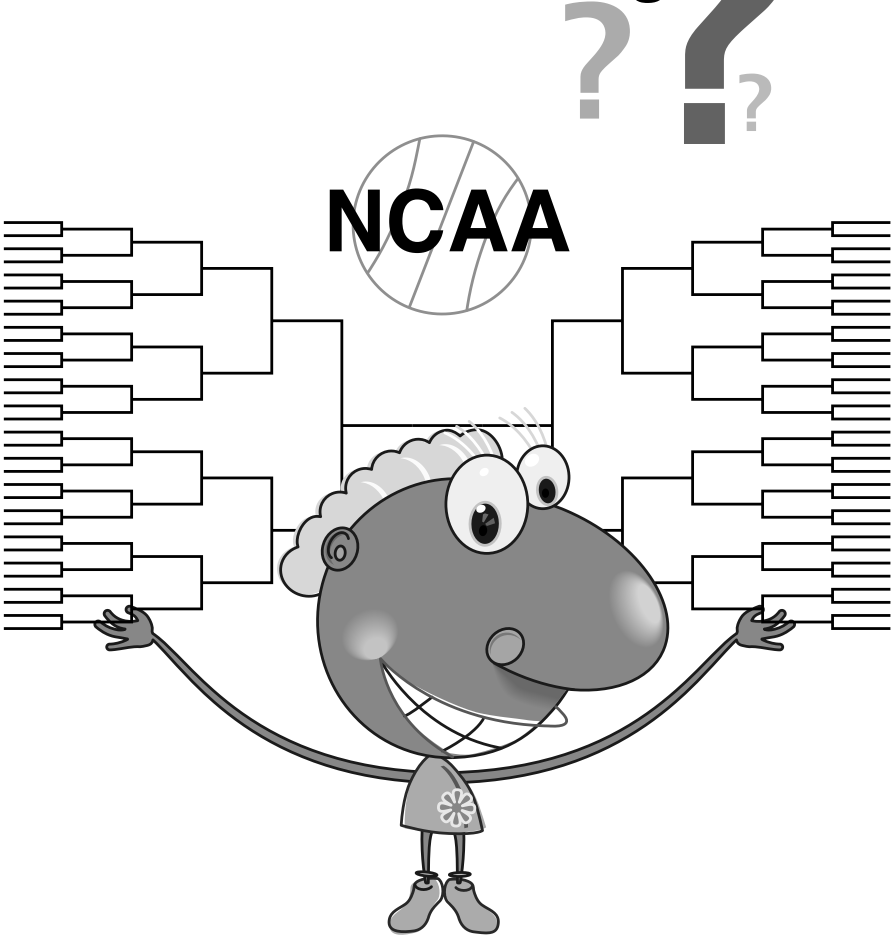


Figure This! In March, the National Collegiate Athletic Association (NCAA) holds a basketball tournament. As soon as a team loses one game, it is out of the tournament. The last unbeaten team is the winner. There were 64 teams in the 1999 men's tournament. In 1985, there were only 32 teams. How many more games were played in the 1999 tournament than in 1985?

Hint: When the final winner is determined, how many teams will have lost?

Thinking about a problem from different perspectives and looking for a pattern are powerful problem-solving strategies. Such strategies are important tools for planning space missions, tournaments, and making business decisions.

Answer: 32 games

Figure This!

Get Started:

Think about the problem in terms of losses as in the hint, or think about how many games there will be and make a schedule. Start with two teams; then three teams. Continue adding more teams and look for patterns. Use the pattern to determine the numbers of games in 1985 and 1999.

Complete Solution:

There is more than one way to do this problem.

- Consider that every team except the winner loses exactly 1 game. If there are 64 teams in the tournament and 1 winner, then there were 63 losing teams. This means there were 63 games. If there are 32 teams in the tournament and 1 winner, there were 31 games. So, 63 – 31 = 32 games.
- Create a table like the one below to determine the number of games required for a 64-team tournament.

ROUND	NO. OF TEAMS AT THE START OF THE ROUND	NO. OF GAMES	NUMBER OF WINNERS
1	64	32	32
2	32	16	16
3	16	8	8
4	8	4	4
5	4	2	2
6	2	1	1
		TOTAL NUMBER OF GAMES	63

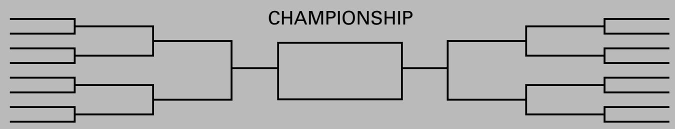
Add the numbers for the games in the table to find the total number, 63, of games for 64 teams. A similar table can be used to find that a 32-team tournament requires 31 games. The difference is 63 – 31 = 32.

- After the first round of 32 games in 1999, the number of teams was the same as in the 1985 tournament. So, the difference in the number of games is the number of games played in the first round, 32.
- Another way is to consider the number of games necessary with 2 teams, then 3, and so on. If you can identify a pattern, you can use that pattern to determine the number of games for 32 and 64 teams.

# of TEAMS	1ST ROUND	2ND ROUND	3RD ROUND	# of GAMES
2 teams (A and B)	A plays B			1 game
3 teams (A, B, C)	A plays B C does not play	B plays C		2 games
4 teams (A, B, C, D)	A plays B C plays D	B plays D		3 games
5 teams (A, B, C, D, E)	A plays B C plays D E doesn't play	B plays D E still doesn't play	B plays E	4 games

As shown in the table, in each case, there is one less game than the number of teams. A 64-team tournament would require 63 games to determine a winner, while a 32-team tournament would require 31 games. The difference is 63 – 31, or 32 games.

- A graphical method to solve the problem uses tournament brackets showing how the teams are scheduled to play. Consider the brackets below.



With this set of brackets, you see that half of the teams are grouped in pairs on the right and half on the left to begin. The first round is played in the outside brackets for a total of 16/2 or 8 games. The next round with teams paired in the second set of brackets consists of 8/2 or 4 games. Continuing this process shows that for 16 teams, there are 8 + 4 + 2 + 1, or 15 total games. Using brackets can be done for any number of teams. For 64 teams, there are 63 games; for 32 teams, 31 games. 63 – 31 = 32.

Try This:

- Find out how tournaments involving your school are scheduled.

Additional Challenges:

1. In 1983, there were 48 teams in the NCAA tournament. The top 16 teams did not have to play in the first round. How many games were played to determine the champion?
2. In a double-elimination tournament, a team is eliminated when it loses two games. If there are 8 teams in a double-elimination tournament, what is the maximum number of games required to determine a champion?
3. Why do you think that the number of teams invited to compete in a tournament is usually a power of 2 (2, 4, 8, 16, and so on)?

Things to Think About:

- How are tennis tournaments scheduled?
- Some tournaments, such as Major League Baseball's World Series, are won by the first team to win four games. How many games are usually played in the World Series?
- Chess competitions often use ladder tournaments. How do you think a ladder tournament might work?

Did You Know That?

- The NCAA Championship game first appeared on television in 1962 when an edited version was shown on ABC's Wide World of Sports.
- The first NCAA Championship for women's basketball was played in 1982.
- Oregon won the first NCAA championship in men's basketball, held in 1939.
- During the first 12 years of the men's tournament, only 8 teams played. As the tournament gained in importance, the field was expanded to 16 teams, then to 32, and then to its present size of 64.

Resources:

Book:

- Halmos, Paul R. *Problems for Mathematicians Young and Old*, Dolciani Mathematical Expositions Number 12. Washington, DC: Mathematical Association of America, 1991.
- Hillstrom, Kevin, Laurie Hillstrom, and Roger Matuz. *The Handy Sports Answer Book*. Farmington Hills, MI: Visible Press, 1998.
- *Sports Illustrated 1999 Sports Almanac*. New York: Little, Brown and Co., 1998.
- Wolf, Alexander. *Sports Illustrated 100 Years of Hoops*. Birmingham, AL: Oxmoor House, 1991.

Website:

- **http://www.ncaa.org/**
- **http://forum.swarthmore.edu/dr.math/problems/topp.8.17.96.html**

Answers to Additional Challenges:

(1.)
47 games.

(2.)
15 games.

(3.)
Using powers of 2, the pairings in each round come out even. Therefore, no teams must sit out.

Notes:

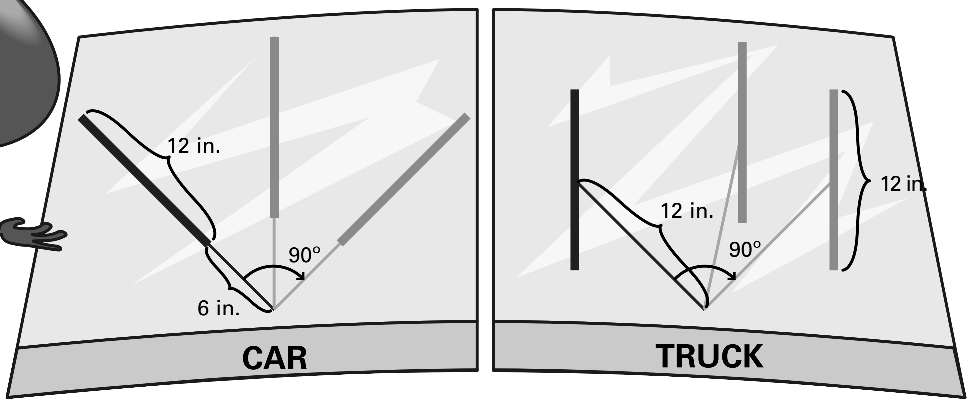
Math Challenge #32 WINDSHIELD WIPERS

FigureThis! Math Challenges for Families

IT'S RAINING!!

Who sees more: the driver of the car or the truck?

FigureThis! Which wiper cleans the greater area?



Hint: Draw models of the areas, then cut them out and compare.

Areas of familiar geometric shapes can be used to find the areas of other shapes. Surveyors, carpet layers, designers, and building contractors all calculate areas in their work.

Answer:
The car wiper cleans a greater area.

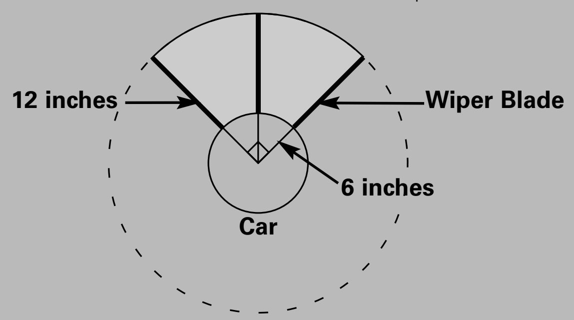
Figure This!

Get Started:

Draw the area cleaned by each wiper. What shape is each area? How could you find the area of each shape?

Complete Solution:

The car wiper rotates through 1/4 (or one quarter) of a circle.The area cleaned is the difference in the areas of two quarter-circles as shown.



The area of a circle can be found using the following formula:

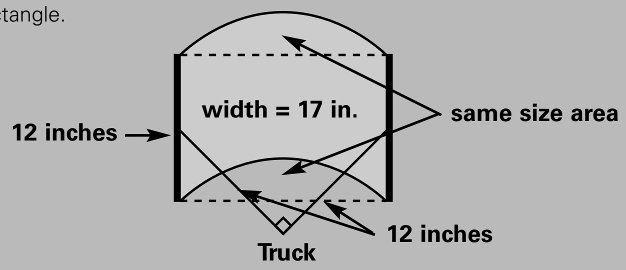
$$\text{Area} = \pi \bullet \text{radius} \bullet \text{radius or } A = \pi r^2$$

The wiper arm swings through an arc of 90˚, or 1/4 of a circle. The area of a quarter-circle can be found by multiplying the area of the corresponding circle by 1/4. In this case, the area cleaned by the wiper is the area of the bigger quarter-circle minus the area of the smaller one. The radius of the big circle is 6 + 12, or 18 inches. The radius of the small circle is 6 inches so that the area cleaned is:

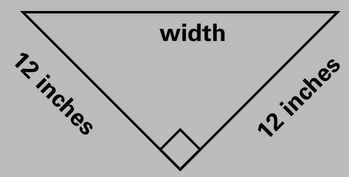
$$1/4\,(\pi \bullet 18^2) - 1/4\,(\pi \bullet 6^2) = 72\pi$$

π is about 3.14 so 72 • π is about 226 square inches.

The area of the shape cleaned by the truck wiper is really the same as a rectangle.



The curved area at the top of the shape is the same size as the part at the bottom that is not cleaned by the wiper. The height of the rectangle is determined by the size of the wiper: 12 inches. The width of the rectangle can be found by drawing the figure to scale, then measuring, or by using the Pythagorean theorem. According to the Pythagorean theorem, the sum of the squares of the two shorter sides of a right triangle equals the square of the longest side.



$$\text{width}^2 = 12^2 + 12^2 \qquad \text{width}^2 = 288$$

Therefore, the width of the rectangle is the square root of 288, or approximately 17 inches.The area of a rectangle can be found by multiplying its height and its width. In this case, the area is about 17 • 12, or 204 square inches. This is less than the area cleaned by the car wiper.

Try This:

Use straws to make a model of each of the wipers. Move each wiper arm 90° and draw the shape that the wiper blade describes.

Additional Challenges:

1. Cut a parallelogram as shown from a sheet of paper.

   Make a rectangle from the parallelogram using only one cut and rearranging the parts.

2. Cut a triangle from a sheet of paper. Cut the triangle into three pieces and put the pieces together to make a rectangle.

Things to Think About:

- Can you design a 12-inch wiper that would clean more area than either of the two types in the challenge?
- Look at some cars, trucks, and other vehicles. How are their windshield wipers designed? How many different designs can you find?
- Which do you think would clean a windshield better: one large wiper or two smaller wipers that overlap?
- How do rear-window wipers differ from windshield wipers?
- Why do some Mercedes use only one big wiper instead of two?

Did You Know That?

- Most wipers are between 16 and 22 inches long and sweep out an angle between 90° and 120°.
- Robert William Kearns, the developer of the intermittent windshield wiper, has spent much of his life fighting for the rights of inventors. He had to go to court to secure rights to his invention.

Resources:

Website:

- **www.nwb.co.jp/e/encyclo/history.html**

Answers to Additional Challenges:

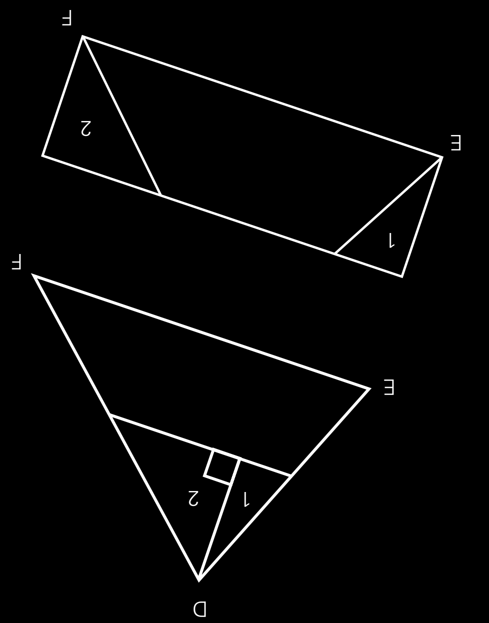
(1) :

One way is to cut a right triangle off one end of the parallelogram, then tape it to the other end of the remaining figure. Both the original parallelogram and the new rectangle have the same area.

(2) :

One way is to cut the triangle as shown below.

- Turn triangle 1 to fit D on E.
- Turn triangle 2 to fit D on F.



Notes: