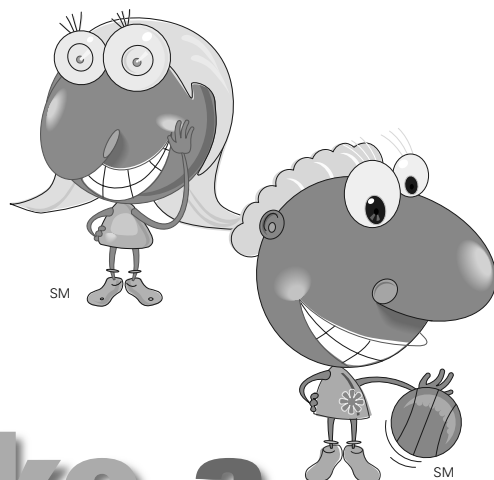


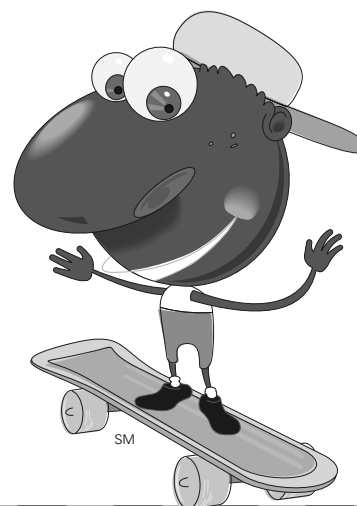
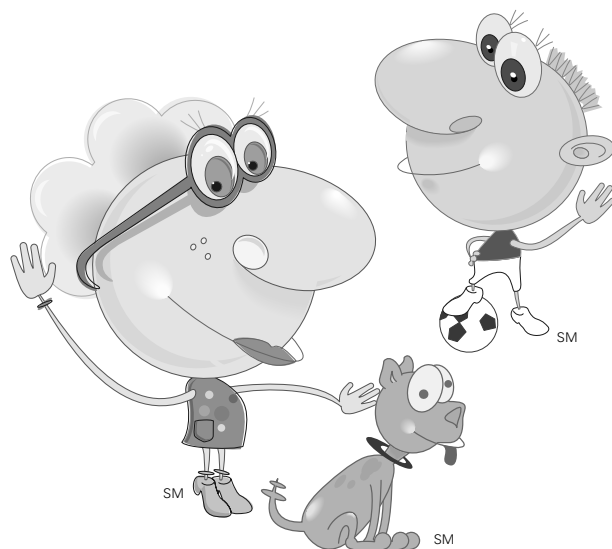
FigureThis!

Math Challenges for Families



Take a Challenge!

Set V: Challenges 65 - 80





Axis

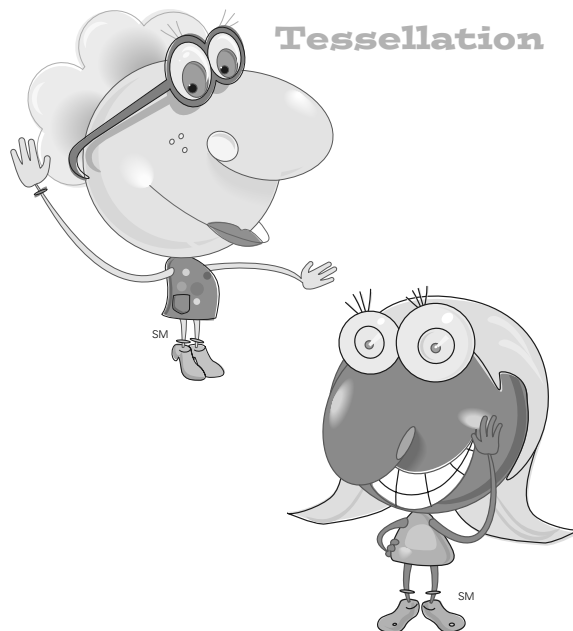
Thank you for your interest in the **Figure This! Math Challenges for Families**. Enclosed please find Challenges 65 – 80. For information about other challenges, go to **www.figurethis.org**.

The **Figure This!** Challenges are family-friendly mathematics problems that demonstrate what middle-school students should be learning and emphasize the importance of high-quality math education for all students. This campaign was developed by the National Action Council for Minorities in Engineering, the National Council of Teachers of Mathematics, and Widmeyer Communications, through a grant from The National Science Foundation and the US Department of Education.

Figure This!

Math Challenges for Families

Tessellation

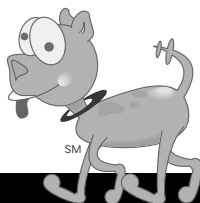


Polygon

Ratio



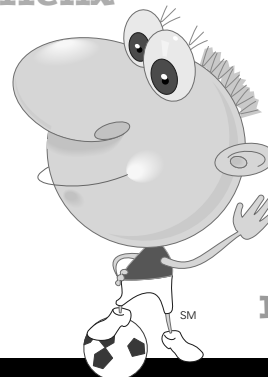
Tangent



Perimeter



Helix



Exponent



We encourage you to call toll free **1-877-GO-SOLVE** or visit our website at **www.figurethis.org** where you can find these and other challenges, along with additional information, math resources, and tips for parents.

Figure This!

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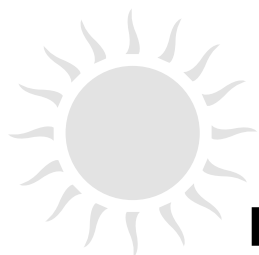


Figure This!
Math Challenges for Families

How l o n g before you
burn?

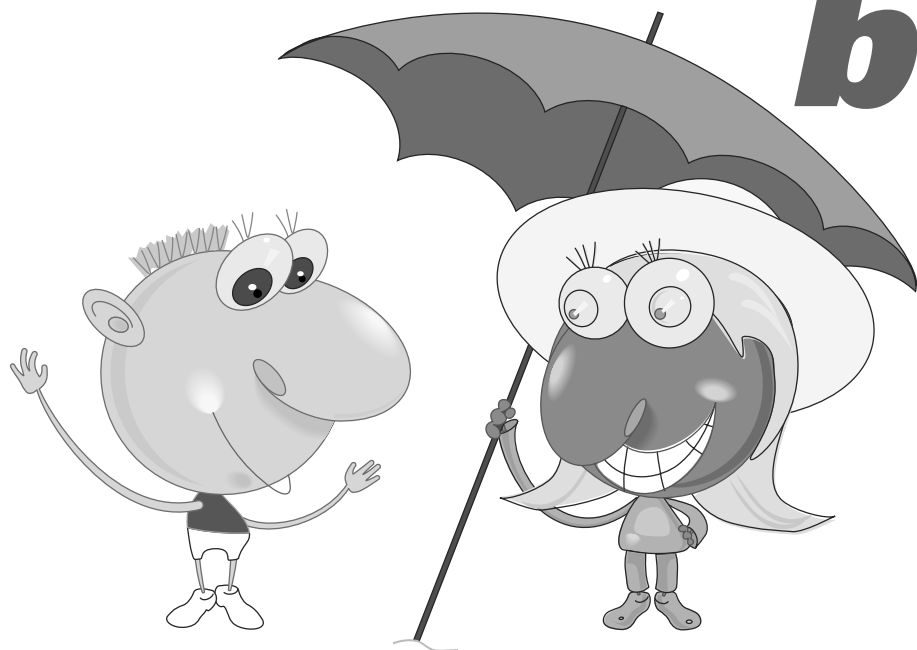


Figure This! Helix can usually stay in the sun 8 minutes before being sunburned. Using a sun tan lotion product with SPF (Sun Protection Factor) 10 means that he can stay in the sun 8×10 , or 80 minutes before being burned. Helix put SPF 10 lotion on five minutes after he got to the beach. An hour later, he is burned. Why?

Hint: How much burning time was gone before the lotion was used?

Citizens need to understand what numbers represent in situations in their daily lives. Understanding such numerical values as SPF, the category of a hurricane, and electrical wiring codes is important for making informed decisions.

Since Helix was exposed to the sun for 5 minutes without protection, he had only 3 minutes left before he would burn. A sunscreen with an SPF of 10 would extend the time left to 30 minutes.

Answer:

Figure This!

Get Started:

How long was Helix at the beach before applying the sunscreen? How many more minutes of exposure could his skin withstand before burning with sunscreen? with no sunscreen?

Complete Solution:

An SPF 10 product used before going in the sun, and when the user normally burns in 8 minutes, provides about 8×10 , or 80 minutes of protection. Putting the lotion on 5 minutes after being in the sun uses $\frac{5}{8}$ of the time before being burned. Thus, $\frac{3}{8}$ of the normal protection time before burning is left. Because $(\frac{3}{8})(80)$ is 30, a person has 30 minutes before burning.

Note: The limits of safe exposure to sunlight vary by individual, season, and geographic location.

Try This:

- Do some research on the web to see the effects of extended exposure to sunlight.
- Determine what type of sun protection you should use.
- Find out what ultraviolet (UV) rays are and how they are measured.
- Consult a doctor, nurse, or pharmacist about potential allergies to certain sunscreens.

Additional Challenges:

(Answers located in back of booklet)

1. The chart below shows skin color and recommended SPF to avoid sunburn. Suppose there was another complexion called “extremely fair,” what SPF would be recommended for a person with that complexion?

Skin Color / Complexion	Recommended SPF to Avoid Sunburn
Dark	2
Medium	6
Light	15
Very Fair	30

2. A microwave oven can bake a potato in 5 minutes. To crisp the skin, a potato can be transferred to a regular oven set at 350° after 3 minutes in the microwave. If it normally takes 1 hour for a potato to bake in the regular oven, when will the potato be done?
3. Water flows into a pool spa at the rate of 14 gallons per minute. If the pool is one-third full and it holds 360 gallons, how long does it take to complete filling?

Things to Think About:

- How long can you usually stay in the sun before getting sunburned?
- What type of sun protection do you use? Why should you re-apply suntan lotion during the course of the day?
- In which parts of the world are severe sunburns most likely?
- Can you sunburn on a cloudy day?
- Would an SPF of 200 make any sense?
- Why do people sunburn faster when they are playing on the water, on snow, or at the beach?

Did You Know That?

- A tan is your skin’s reaction to the damage done by ultraviolet radiation.
- Infrared radiation from the sun also can be harmful to the skin, increasing the risk of skin cancer and causing premature aging.
- In North America, the peak hours of exposure to ultraviolet radiation are between 10:00 A.M. and 3:00 P.M.
- The exposure to ultraviolet rays increases at higher altitudes and in regions where the ozone layer is thinner.
- Many medicines, including certain antibiotics, can increase a person’s sensitivity to sunlight.
- The chemicals in some perfumes also can heighten sensitivity to sunlight.
- Symptoms of sunburn—such as reddened skin—can take up to 24 hours to appear.

Resources:

Magazine:

- Crabtree, K. “The Great Cover-up.” *Ski* (February 2001): 94–96.

Websites:

- www.htropic.com/sunfacts/index.shtml
- www.emedicine.com/EMERG/topic798.htm
- www3.utsouthwestern.edu/library/consumer/sunbrn.htm



FigureThis!
Math Challenges for Families

Why are airport runways numbered?

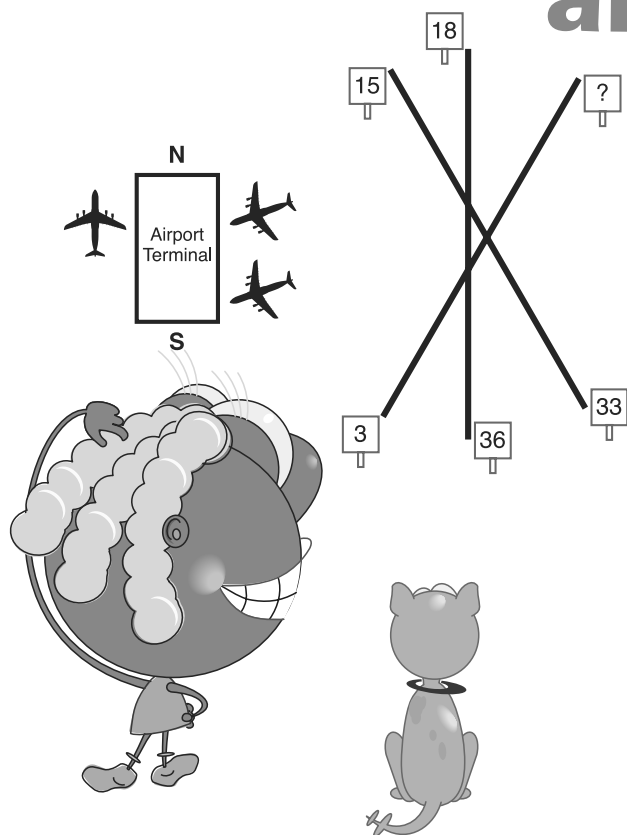


Figure This! Airport runways are labeled with numbers at both ends. The pilot sees the signs when the plane is landing. What is the missing runway number?

Hint: Place a zero after each runway number; then think in terms of degrees. The measure of an angle along a straight line is 180° .

Navigation depends on knowing location and heading. Airplane and ship pilots, surveyors, explorers, and scouts use compasses to find directions.

Figure This!

Get Started:

A heading of 10° means you are traveling in a direction 10° clockwise from due north. The number on the corresponding runway would be 1. Which heading corresponds to a runway labeled 19?

Complete Solution:

- The numbers painted on each end of a runway tell a pilot the compass direction in which to land the plane. For example, consider a runway with ends that face north and south. The north end is labeled with the number 18. Placing a 0 after the 18 to make 180 indicates that a plane landing at this end would be heading south at 180° .
- In the challenge, the runway with the missing number is labeled 3 at its opposite end. The number 3 indicates a heading of 30° . Since each runway is a straight line, the difference between the headings for its two ends is always 180° . So, if one end requires a heading of 30° , the other corresponds to a heading of $30^\circ + 180^\circ$, or 210° . The runway sign, therefore, should be 21.

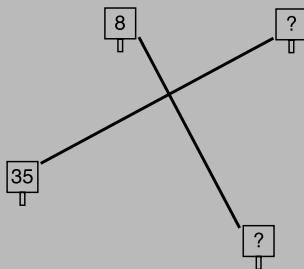
Try This:

- Research websites to learn how the runways are arranged at several different airports. How are the runways numbered at your local airport?
- Identify a building or object that is due north of your home.
- Estimate the compass headings for the street or road nearest your home.

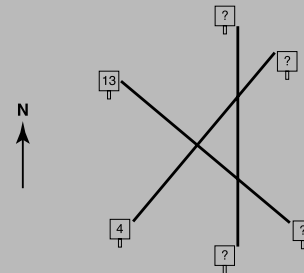
Additional Challenges:

(Answers located in back of booklet)

1. The diagram below shows the runways at a small airport. Use the information given to find the missing runway numbers.



2. Determine the missing runway numbers for the airport shown below.



3. A plane has a 10° heading, turns and goes in the opposite direction. What is the new heading?

Things to Think About:

- Most airports design their runways so that planes are able to take off and land into the wind as much as possible.
- Why are many paved runways made out of concrete and not asphalt?
- How do the surroundings influence where airports are built?
- What length of a runway is required to land a small plane? To land a space shuttle?
- Why do airport runways use 36 instead of 0?

Did You Know That?

- Runways also are labeled with vertical lines, to indicate their length. Each vertical line represents 1000 ft.
- Taxiways—the connections between runways and the terminal building—are usually built at 30° angles to the runways. This allows planes to turn from one to the other without coming to a stop.
- Taxiways are labeled with letters, to indicate the different paths from runways to terminals.
- O'Hare International Airport in Chicago, Illinois, is one of the busiest airports in the world, with more than 900,000 take-offs and landings per year.
- The United States has more than 18,000 airports. More than 8000 have paved runways, while many of the others have grass runways. Approximately 4900 have lighted runways.
- Parallel runways are labeled R and L. For example, two north-south runways might be labeled R18-36L and L18-36R.
- Most interstate highways in the United States are numbered according to the following system: east-west highways are even-numbered, with the numbers increasing from south to north; north-south highways are odd-numbered, increasing from west to east.

Resources:

Book:

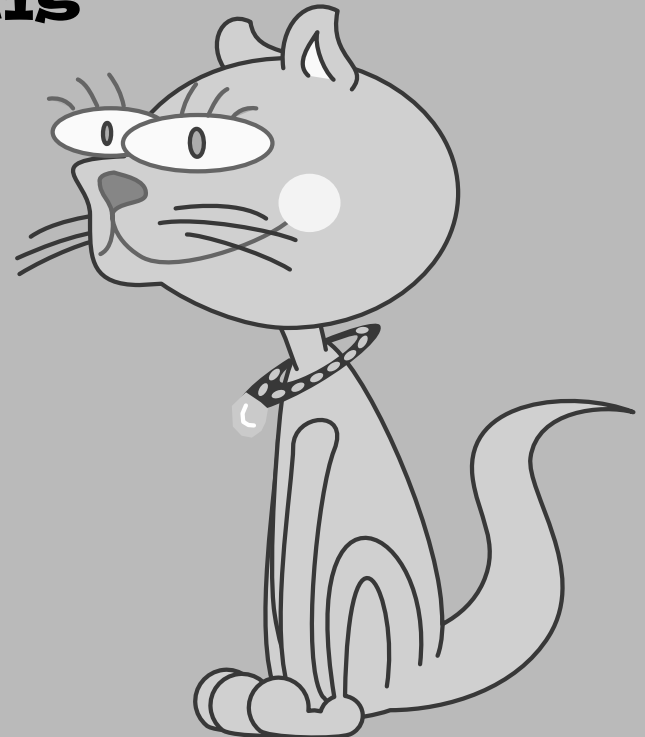
- Billstein, Rick. "You Are Cleared to Land." *Mathematics Teaching in the Middle School* 3(May 1998): 452-456.

Websites:

- Most major U.S. airports have their own websites.
- www.boeing.com/assocproducts/noise/fairbanks.html

Notes:

Axis





FigureThis!
Math Challenges for Families

Does Polygon need a jacket?

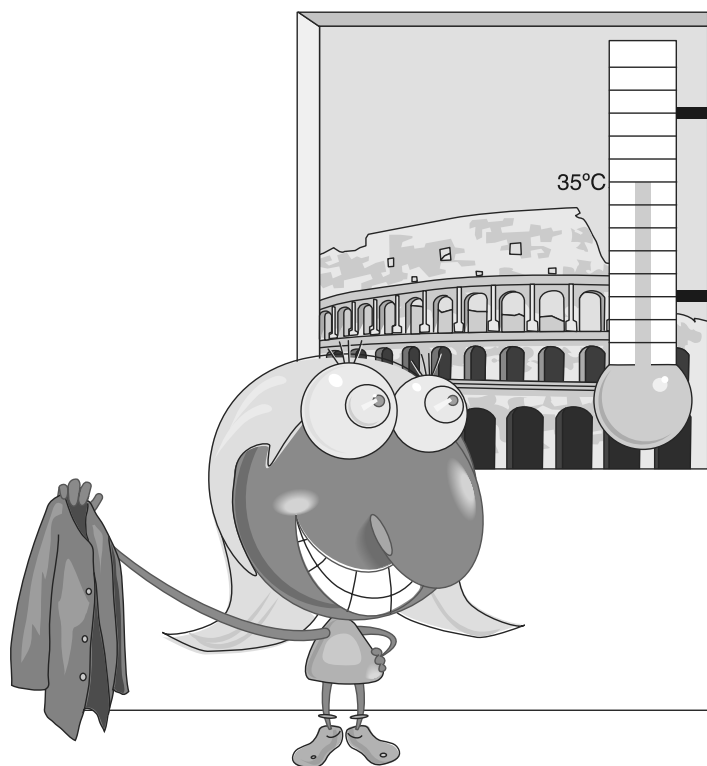


Figure This! Before leaving her hotel, Polygon noticed that the outside temperature was 35° Celsius. Does she need a jacket for the day?

Hint: On the Celsius scale, water freezes at 0° and boils at 100°. On the Fahrenheit scale, water freezes at 32° and boils at 212°.

Having benchmarks in metric units of measurement is important in a global economy. Scientists, doctors, nurses, economists and farmers all need to have a working knowledge of international measuring systems.

Polygon does not need a jacket, since 35° C is 95° F.

Answer:

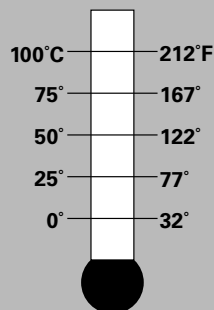
Figure This!

Get Started:

Sketch two thermometers side by side. Label one with the freezing and boiling points in degrees Celsius, the other with the same points in degrees Fahrenheit. Then determine the temperatures halfway between freezing and boiling on each thermometer. A change in degrees Celsius is proportional to the corresponding change in degrees Fahrenheit.

Complete Solution:

- In the diagram on the Celsius thermometer, the halfway mark is 50°, and the quarter marks are 25°, 50°, and 75° since there are 100 divisions between 0° and 100°. On the Fahrenheit thermometer, there are 180 divisions between 32° and 212°. One half of 180 is 90 and 90+32=122, halfway between. Similarly, the quarter marks are 77°, 122°, and 167°. The half and quarter marks on the scales should match. A temperature of 35° C is well above 77° F so Polygon would not need a jacket.



- To determine the exact temperature, use the formula,

$$F = \left(\frac{9}{5}\right)C + 32.$$

When C = 35°, the temperature is 95° F, and Polygon does not need a jacket.

Try This:

- Examine newspapers, magazines, and other media reports to see how the temperature is measured in different cities around the world.
- Look in your cupboards to see how metric labels are used.
- The speedometers in many cars show speed in miles per hour and in kilometers per hour. What would your local speed limits be in kilometers per hour?

Additional Challenges:

(Answers located in back of booklet)

- Should Polygon wear a jacket if the temperature is 10° C?
- A formula for converting between temperatures in Fahrenheit (F) and Celsius (C) is:

$$C = \frac{5}{9} (F - 32)$$

What is the temperature in degrees Celsius when it is 86° F?

- Some scientists have observed a connection between the air temperature and the frequency with which a cricket chirps. If you count the number of times a cricket chirps in 1 minute, divide by 4, then add 37, the result is a good estimate of the temperature in degrees Fahrenheit. If a cricket chirps 120 times in 1 minute, how warm is it?
- At what temperature would a Celsius thermometer and a Fahrenheit thermometer give the same reading?
- If 50 mph is about 80 km/h, would you be speeding at 35 km/h in a 25 mph zone?

Things to Think About:

- What is a comfortable temperature for bath water?
- Is the body temperature of a fish the same as the temperature of the surrounding water?
- Why do scientists prefer the metric system to the U.S. conventional system?
- Why has the American public resisted the adoption of the metric system?
- Why is the so-called “normal” body temperature of 98.6° F reported to the tenth of a degree?

Did You Know That?

- One easy rule for estimating temperature in Fahrenheit is to double the temperature in Celsius and add 30°. Another simple method is to remember this poem by Carole Greenes, Boston University:

30° is hot,

20° is pleasing,

10° is not,

0° is freezing.

- A meter is a “big yard,” while a liter is a “big quart.”
- A ton is 2000 lb. A metric ton is 1000 kg. The difference is 200 lb or about 91 kg.
- At sunset, the temperature at the moon’s equator is about 58° F. During the night, it drops to about –261° F.
- The highest temperature ever recorded in the United States was 134° F on July 10, 1913, at Death Valley, California.
- German physicist Daniel Gabriel Fahrenheit (1686–1736) invented the mercury thermometer in 1714. Before then, thermometers contained alcohol.
- Swedish astronomer Anders Celsius (1701–1744) introduced his scale in 1742. The Celsius scale has become part of the metric system of measurement and is used throughout the world.

- British mathematician William Thomson, Lord Kelvin (1824–1907), introduced a scale that begins with absolute zero, the temperature at which all motion of atoms stops. For comparison, 0°K is -273.16°C .
- The different formulas given above for Fahrenheit and Celsius conversions are equivalent.

Resources:**Book:**

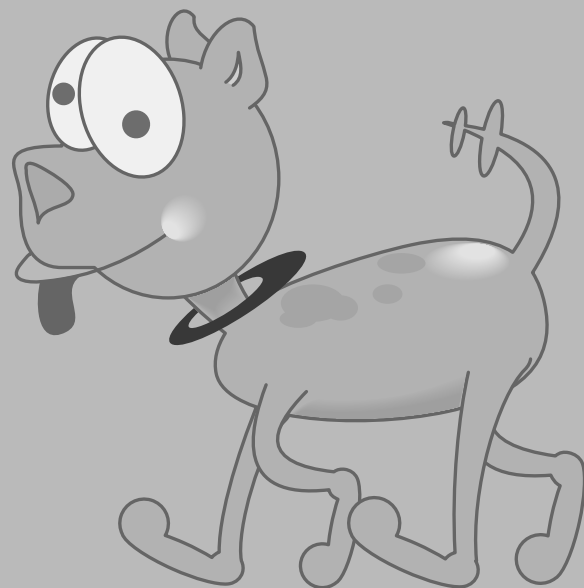
- *The Guinness Book of World Records*. New York: Bantam Books, 1998.

Websites:

- www.usatoday.com/weather/wtempcf.htm
- www.athena.ivv.nasa.gov/curric/weather/fahrcels.html

Notes

Tangent





FigureThis!
Math Challenges for Families

Does drinking soda affect your health??

"Teenage girls' soda intake is linked to broken bones"

	Fractures	No Fractures
Drank Cola	38	69
Did Not Drink Cola	5	52

Source: USA Today, Thursday, June 15, 2000.

Figure This! For this study, researchers questioned ninth- and tenth-grade girls at a Boston-area high school. Do the data support the headline?

Hint: What percentage of the teenage girls who drank cola also had fractures?

Data organized in tables help people understand how different categories or groups are related. Tables are used by researchers, businesses, media, and the government to display information about study results, profit and loss, news stories, and census data.

Based on the information given, the headline is believable.

Answer:

Figure This!

Getting Started:

If there is a connection between soda intake and broken bones, you might expect that teenagers who drink cola would be more likely to have fractures than teens who do not drink cola. Use the hint to begin your explorations.

	Had a fracture	Did not have fractures
Drank cola	$\frac{38}{38+69} \approx 36\%$	
Did not drink cola		

Complete solution:

One way to analyze this situation is to consider two groups of teens in the study: those who drank soda, and those who did not. From the table, there are $38 + 69$, or 107 students who drank cola, and $5 + 52$, or 57, who did not. Of the 107 students who drank cola, 38 had fractures, or approximately 36%. Of the 57 students who did not drink cola, 5 had fractures, or about 9%. The information is summarized in the table.

	Had a fracture	Did not have fractures
Drank cola	$\frac{38}{38+69} \approx 36\%$	$\frac{69}{38+69} \approx 64\%$
Did not drink cola	$\frac{5}{57} \approx 9\%$	$\frac{52}{57} \approx 91\%$

Based on this information, students who drink cola seem to be more likely to have fractures. Just because an association exists, however, does not mean that drinking cola causes fractures. Other factors may be involved.

Try This:

- Make a small circle with your thumb and index finger. Hold the circle a full arm's length away. Look through this circle and focus both eyes on an object some distance away. Shut one eye at a time. The eye through which the object remains in the circle is your dominant eye. Survey 30 people about both their dominant eye and their dominant hand (right or left). Record your results in a table. Use the results to describe the association between eye and hand dominance.
- Look for reports of medical studies in the media. If possible, display the data given in a table. Do the data seem to support the claims made in the report?

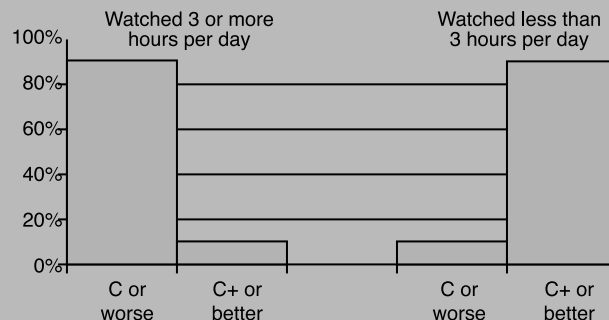
Additional Challenges:

(Answers located in back of booklet)

1. Which of the following bar graphs supports the argument that students who watch less television earn better grades?

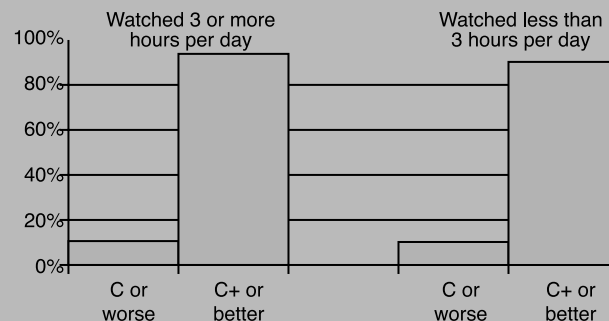
a.

Television and Grades



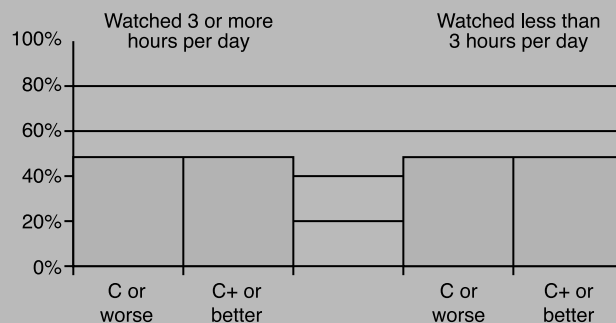
b.

Television and Grades



c.

Television and Grades

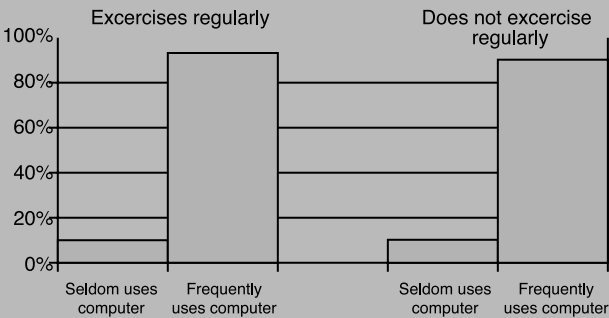


2. In 1999, the two top-earning movies were “Titanic” and “Star Wars, Part IV.” Helix surveyed 200 people who had seen both movies. He recorded the data in a table.

	Liked “Titanic”	Disliked “Titanic”	Totals
Liked “Star Wars”	70		
Disliked “Star Wars”	50	37	
Totals			200

- a. Complete the table above.
- b. Based on this information, do you think that someone who liked “Titanic” would also like “Star Wars, Part IV”?
3. This graph represents survey results about the relationship between the amount of time teen-agers exercise and the amount of time they spend on a computer.

Computer Use and Exercise



What does the graph tell you about the relationship between exercise and computer use?

Things to Think About:

- What factors other than soft-drink intake might affect the frequency of bone fractures in teens?
- Is it easier for you to understand and work with numbers when they are embedded in a paragraph or displayed in a table?
- If there is a strong association between two events, such as hand and eye dominance, does it mean that one causes the other?
- Tables of data as the one given in question 2 of the additional challenges are sometimes called “two-way tables.” For how many cells must you know the entries before you can calculate the remaining values?

Did You Know That?

- Association between events can be positive or negative. Negative association shows that the more of one event, the less of the other, while positive association shows the more of one event, the more of the other.
- To determine if two events are associated, statisticians sometimes use a mathematical technique known as a chi-square test.
- Another tool for organizing information or numbers is a matrix. Matrices look like rectangular tables but typically do not use lines to frame each cell.
- Information that places something in a given category—such as eye color—is called categorical data. Information that can be ordered on a number line—such as height—is called quantitative data.

References:

Book:

Hopfensperger, P., Kranendonk, H., and R. Scheaffer. *Probability Through Data*. Dale Seymour Publications, 1999.

Websites:

- www.amstat.org/publications/jse/v2n2/datasets.rossman.html
- www.lib.msu.edu/behm/readmedlit/hypothesis.htm

Notes:



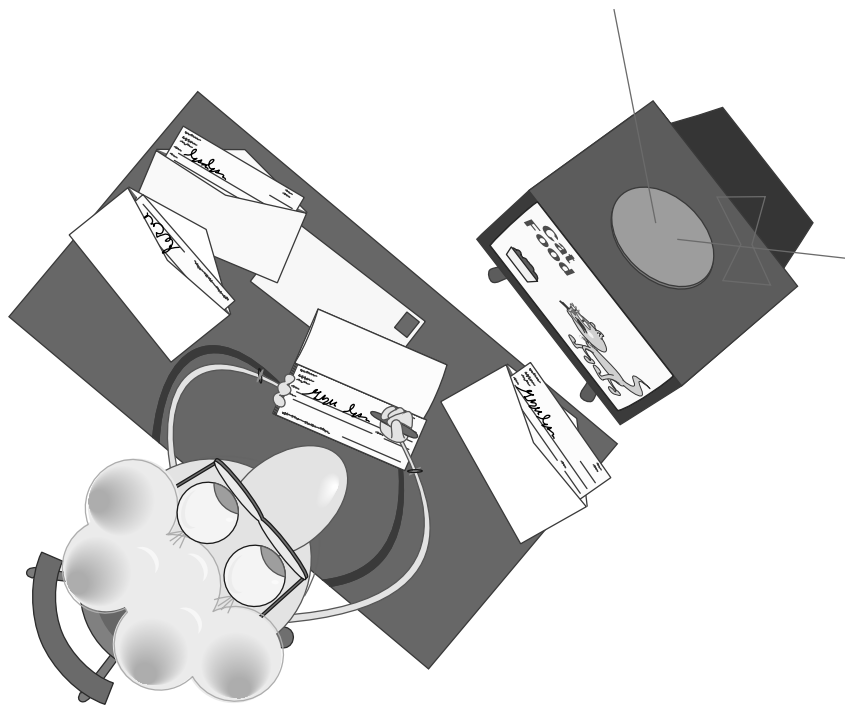
FigureThis!
Math Challenges for Families

How could I send the **check** and **not** pay the **bill**?

Figure This! While watching television, Tessellation placed three checks in three separate addressed envelopes. If she had paid no attention to which check went in which envelope, what is the chance that each check was in the correct envelope?

Hint: Make a list of all the possible outcomes.

Probability is the mathematics of chance. Estimating the chance of rain, the effectiveness of a treatment, and whether a person will have an accident are important to weather forecasters, doctors in diagnosing illnesses, and insurance companies.



The probability is $1/6$.

Answer:

Figure This!

Get Started:

Label the checks *a*, *b*, and *c*, and label the envelopes *A*, *B*, and *C*. Assume check *a* belongs in envelope *A*. In how many ways could check *a* be placed in an envelope? Once *a* is placed, how many choices are there for *b*?

Complete Solution:

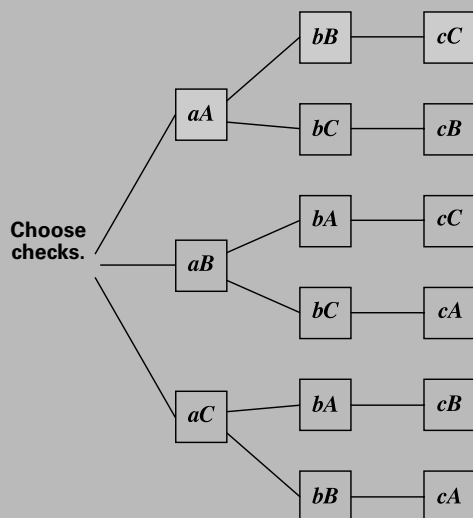
There are several ways to approach this problem.

- The table shows all the possibilities if the checks are labeled *a*, *b*, and *c*, and the corresponding envelopes *A*, *B*, and *C*.

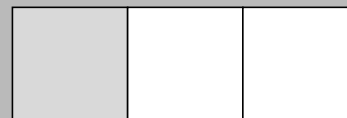
<i>A</i>	<i>B</i>	<i>C</i>
<i>a</i>	<i>b</i>	<i>c</i>
<i>a</i>	<i>c</i>	<i>b</i>
<i>b</i>	<i>a</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>a</i>
<i>c</i>	<i>a</i>	<i>b</i>
<i>c</i>	<i>b</i>	<i>a</i>

There are six different possibilities, each one of them equally likely. Only one, however, has all three checks in the matching envelopes. Since exactly 1 case out of 6 has the checks in the correct envelopes, the probability is $1/6$.

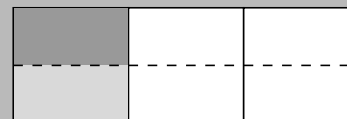
- Another way to solve this problem is draw a tree diagram. In the following diagram, *aA* indicates that check *a* is in envelope *A*. Like the table shown earlier, the tree diagram indicates that there is only 1 correct outcome out of 6 equally likely possibilities. Therefore, the probability of the correct placement is $1/6$.



- You also can consider the problem geometrically. Draw a rectangle like the one below to represent all the possibilities in this situation. Because the chance that the first check is placed in the correct envelope is 1 in 3, shade $1/3$ of the rectangle to indicate this outcome.



If the first check is placed correctly, there are only two possible envelopes for the second check. This means that the chance that the second check is placed in the correct envelope is 1 in 2. To indicate this outcome, darken $1/2$ of the previously shaded area.



If the first two checks are placed correctly, then the final check also must be placed in the correct envelope. No further shading is necessary. The darker shading represents $1/6$ of the entire rectangle, so the probability that all three checks will be placed correctly is $1/6$.

Try This:

- Label four pieces of paper *a*, *b*, *c*, and *d* and four envelopes *A*, *B*, *C*, and *D*. Turn all the items over and mix them up. Then, without reading the labels, place a piece of paper into each envelope. Did you place all four in the right envelopes? Predict how many times you would place all four correctly if you repeated the experiment 29 more times. Then test your prediction by doing the experiment again.
- Draw a large circle on a sheet of paper and divide it into four equal-sized sections (quarter circles). Label each section. Bend a small paper clip into a pointer. Place the pointer at the circle's center. Spin the pointer on the circle. In which section did it stop? Spin the pointer a total of 50 times and record the results. If you displayed your results in a circle graph, what would it look like?

Additional Challenges:

(Answers located in back of booklet)

- If you flip two fair coins, what is the probability that you obtain one head and one tail?
- If you roll one ordinary six-sided die, what is the probability of obtaining a 7?
- What is the sum of the probabilities of all the possibilities in an event?

4. Is the probability of getting a passing grade (at least 60%) by guessing on a 20-question true-false test greater than, less than, or equal to the probability of getting at least 60% correct by guessing on a 10-question true-false test?

5. In the challenge, what is the probability of getting at least one check in the correct envelope?

Things to Think About:

- What is the probability that it will rain in your hometown for seven consecutive days?
- If the probability of an outcome is 0.35, would you expect it to occur exactly 35 times out of 100?
- Does a batting average describe the probability of getting a hit?

Did You Know That?

- Mathematician Abraham de Moivre (1667–1754) was a pioneer in the field of probability. According to legend, the aging de Moivre observed that he needed an extra quarter hour of sleep each day. When he needed 24 hours of sleep, he died.
- Antoine-Nicolas Caritat, Marquis de Condorcet (1743–1794), used probability to argue that capital punishment should be abolished. He reasoned that, however great the probability that each guilty verdict was correct, there remained a small chance of error. As the number of guilty verdicts increased, so would the probability that an innocent person would be executed. Condorcet himself was condemned to death during the French revolution.
- The word *probability* appears to receive its first mention in a 1477 commentary on Dante's *Divine Comedy*.

Resources:

Book:

- Billstein, R., and J. Williamson. *Middle Grades Math Thematics, Book 3*. Evanston, IL: McDougal Littell, 1999.
- Eves, H. *An Introduction to the History of Mathematics*. Philadelphia, PA: Saunders Publishing Co., 1983.

Notes:



FigureThis!
Math Challenges for Families

Does it make a difference? where you shop?

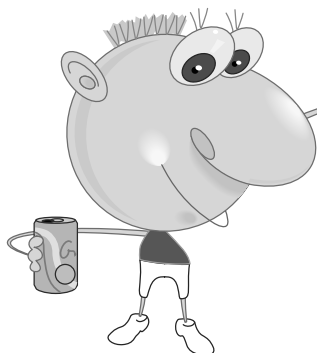
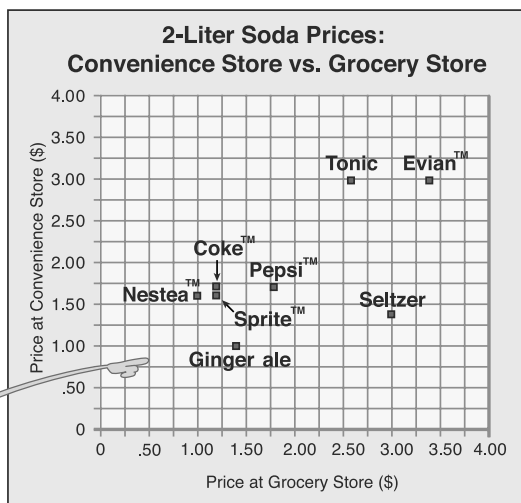


Figure This! Helix checked the prices for two-liter bottles of his favorite soft drinks at two different stores. A graph of the data is shown here. How do the prices compare at the two stores?

Hint: Nestea, the point (\$1.00, \$1.60), costs \$1.00 at the grocery store and \$1.60 at the convenience store. What would the graph look like if the two stores' prices were the same for each item?

Graphs are a visual way to organize and display information, and scatter plots are particularly useful to understand how two quantities are related. Marketing experts, bankers, stockbrokers, demographers, and those in the entertainment business use graphs to understand patterns and trends in their fields.

Half of the items are cheaper at the grocery store. The convenience store's prices seem lower overall, however, due to the big difference in the prices of seltzer.

Answer:

Figure This!

Getting Started:

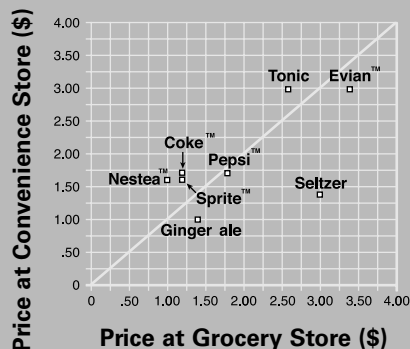
Draw in the line that would show all of the prices equal, the $y = x$ line going from the point (0,0) in the lower left corner to the point (4.00,4.00) in the upper right. What does it mean for a soft drink to be below the line?

Complete solution:

There are several ways to approach this problem.

- Using the hint, draw the line $y = x$ on a copy of the graph. A point below the line indicates that the item's price at the convenience store was less than its price at the grocery store. For example, the point for Ginger ale is below the line. It cost about \$1.40 in the grocery but only \$1.00 in the convenience store. Four of the eight points are below the line, indicating that the grocery store was more expensive for half the items, while the convenience store was more expensive for half the items.

2-Liter Soda Prices: Convenience Store vs. Grocery



- Another way to examine pricing at the two stores uses the vertical distance from each data point to the line $y = x$. On the scatterplot, this distance represents the difference in prices at the two stores. The table below lists the estimated difference for each item in the column for the store where it costs more. The sums of the differences show that prices in the grocery store are higher overall.

Approximate Difference at Most Expensive Store		
Soda	Grocery	Convenience
Nestea™		\$0.60
Coke™		\$0.50
Sprite™		\$0.40
Pepsi™	\$0.00	\$0.00
Seltzer	\$1.60	
Evian™	\$0.40	
Tonic		\$0.40
Ginger Ale	\$0.40	
Total	\$2.40	\$1.90

Try This:

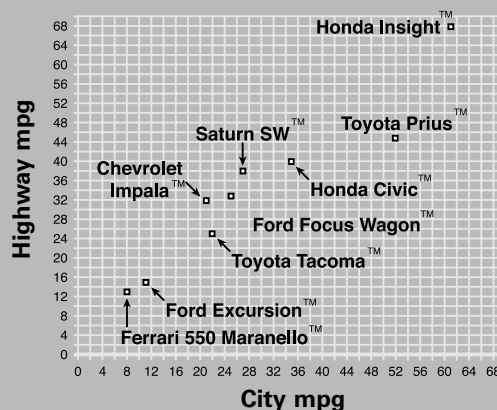
- List eight or nine items you regularly buy in a grocery store; then use the grocery ads in a newspaper for two different stores, or check the prices at the stores to compare their prices.
- Find a scatterplot in a newspaper or magazine. What trends or patterns do you see?

Additional Challenges:

(Answers located in back of booklet)

- The scatterplot shows the estimated mileage per gallon for city driving and for highway driving for the cars as reported in the *USA TODAY* on 2/9/01.

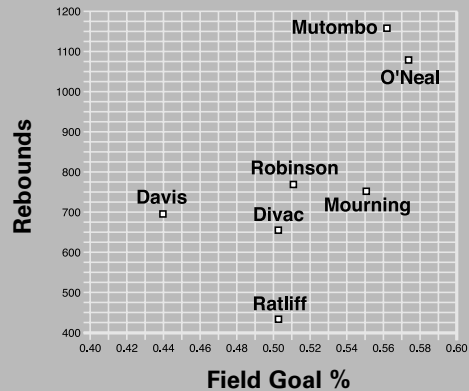
Vehicle miles per gallon



- For which car does it look like the data might be wrong?
 - Which car has the greatest change from estimated mpg in the city to estimated mpg on the highway? the least?
- Without the Toyota Prius, one equation that describes the general relationship between city and highway mileage is: $H = 5.89 + 1.30C$, where H is the highway mpg and C is the city mpg.
 - Use this equation to predict the highway mpg for a Ferrari 456M GT/GTA, which gets about 10 miles per gallon in the city.
 - The actual highway mpg for this car, as reported in the American Council for an Energy Efficient Economy's *Green Book*, is 15. How good was your prediction?

3. The scatterplot below shows some statistics from the 2000 season for seven NBA all-star centers: Antonio Davis, Alonzo Mourning, Theo Ratliff, Shaquille O'Neal, Vlade Divac, David Robinson, and Dikembe Mutombo. Use the information given to rank the players' performances.

2000 Offense and Defense for All-Star Centers



Things to Think About:

- What would the scatterplot in the challenge look like if the two stores had very similar prices?
- What is the leading soft drink, in terms of sales?
- How could your local television station use scatterplots and other graphs?
- How does what you can learn from a table of information compare to what you can learn from a scatterplot?
- Why is the point (0,0) not necessarily a useful place to begin the axes for some graphs?
- Do you think you could tell the difference between two brands of cola in a blind taste test?

Did You Know That?

- Some companies use a *loss leader*—an item advertised at a very low price—to entice consumers into their stores. They hope to make up the loss by selling other goods for higher profits.
- Many of the soft drinks sold in the United States are in liter bottles rather than quarts.
- Several years ago, Coca-Cola™ tried to market their soda in a brown bottle, but consumers did not like it.

Resources:

Books:

- American Council for an Energy Efficient Economy. *Green Book: The Environmental Guide to Cars & Trucks, Model Year 2001*. [This book is available online at www.greenercars.org.]
- *The World Almanac and Book of Facts 2001*. Mahwah, NJ: World Almanac Books, 2001.

Website:

- www.greenercars.org

Notes:

[illegible]



FigureThis!
Math Challenges for Families

Site A! Site B! How alike are we??

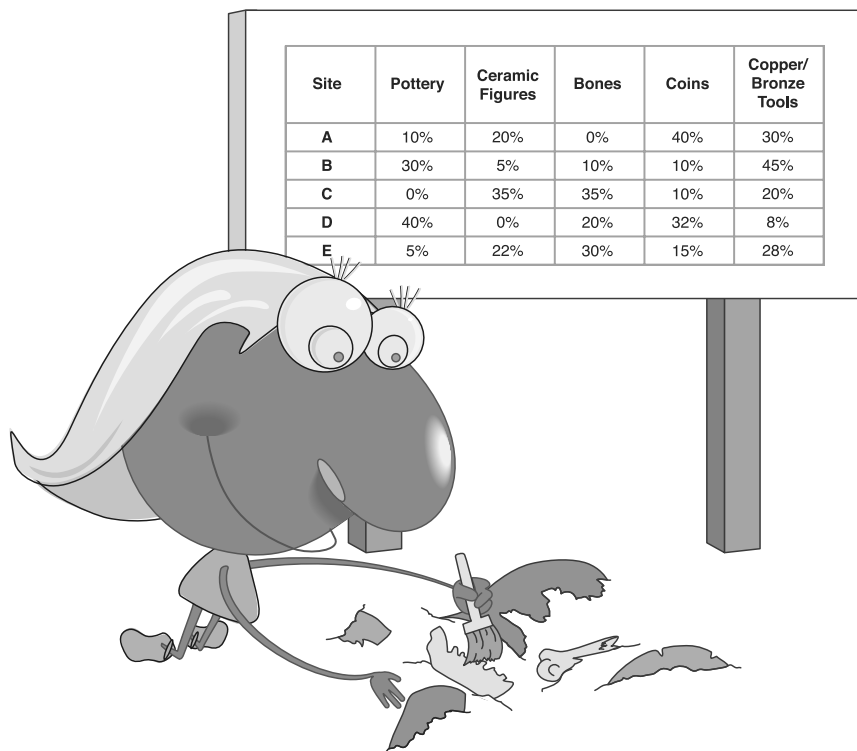


Figure This! Archeologists noticed differences in the types of evidence found at five dig sites. The table shows the percentages of material found at each site. Using this information, which two sites seem to be most alike?

Hint: What is the difference in the percentages of pottery found at sites A and B? What about the percentages of bones?

Numerical criteria can be used to compare objects. Determining likenesses or differences between two sets of information is important for geologists searching for oil, doctors deciding different treatments' effects, and marketing experts deciding on comparable advertising strategies.

Sites C and E have the most similar percentages.

Answer:

Figure This!

Get Started:

Pair the sites and then make a table of the differences found by subtracting the smaller percentage from the larger for each category. For example, using sites A and B, the differences are (30% – 10%), (20% – 5%), (10% – 0%), (40% – 10%), and (45% – 30%). The total of these differences is 20% + 15% + 10% + 30% + 15%, or 90%.

Pair of Sites	Pottery	Ceramic Figures	Bones	Coins	Copper/Bronze Tools	Sum of Differences
A and B	20%	15%	10%	30%	15%	90%

Find the total difference for each pair of sites.

Complete Solution:

Using the hint, find the difference in the percentages of each material at each pair of sites and sum these differences. The resulting information can be arranged in a table as shown. In this table, the cell in row A, column B, corresponds to the sum of the differences for sites A and B, or 90%. (Since there is no difference between a site and itself, the corresponding cells have 0% as entries.)

	A	B	C	D	E
A	0%	90%	100%	100%	64%
B	90%	0%	110%	84%	84%
C	100%	110%	0%	124%	36%
D	100%	84%	124%	0%	104%
E	64%	84%	36%	104%	0%

The pair of sites with the smallest sum (36%) is C and E. This indicates that the percentages of materials at the two sites are the most similar.

Try This:

- Look at the shoes in your house. Identify at least four characteristics that can be used to describe shoes. Categorize the shoes according to these characteristics. Using these criteria, which pairs of shoes are most alike?
- Which of the cereals in your cupboard are most alike in terms of nutritional value?
- Use the web to find other examples of mathematics in the work of archeologists.

Additional Challenges:

(Answers located in back of booklet)

- Look at the table given in the complete solution to the challenge. If you draw a line from the upper left-hand corner of the table to the lower right-hand corner, the entries in the part above the line match those below the line. Why is this true?
- Using data collected from about 20 sites, archeologist T. Patrick Culbert of the University of Arizona estimated that in the past, there were approximately 200 people per square kilometer in the southern lowlands of Central America. How does this figure compare to the most recent population density of Minnesota. (In 2000, the state had a population of 4,775,508 and an area of 79,617 square miles.
Note: 1 square kilometer equals 0.386102 square miles.)
- The table below shows the prices for bread, milk, eggs, coffee, and orange juice at five neighborhood markets. Which two stores seem to have the most similar prices? Which stores seem to differ the most?

Store	Bread	Milk	Eggs	Coffee	Orange Juice
A	\$0.89	\$1.90	\$1.79	\$5.99	\$2.39
B	\$1.19	\$1.65	\$1.75	\$6.24	\$2.49
C	\$1.09	\$1.99	\$1.65	\$5.33	\$3.24
D	\$1.49	\$1.75	\$2.05	\$5.88	\$2.75
E	\$0.99	\$1.89	\$2.13	\$6.12	\$2.33

Things to Think About:

- How can archeologists tell the age of a bone or piece of pottery?
- What do you think the archeologists of the future will find when they excavate your hometown 1000 years from now?
- What causes ancient cities to be destroyed and become buried under land or water?
- How would you compare the climates of several different cities?

Did You Know That?

- Archeologists also explore under water to find buildings and relics from earlier cultures.
- In ancient Rome, it was a mark of status to be buried by a busy road, where passers-by could stop and read the tomb's inscription. This was thought to give the dead person a certain measure of immortality.
- The coins found at archeological sites help historians understand the trade and economic patterns of ancient cultures.

- Archeologists often use dental picks and cotton swabs to scrape away dirt and clay from coins, bones, pottery, and other finds.
- By analyzing differences in embroidery, color, and design, archeologists can sometimes identify the village in which an item of clothing was made.
- Evidence about grid plans for cities, ceremonial avenues, apartment compounds, and plaza structures has led archeologists to think that the several early cities from the Basin of Mexico resemble the ancient Aztec city of Teotihuacan.

Resources:

Books:

- Gardner, M. *Time Travel and Other Mathematical Bewilderments*. New York: W. H. Freeman and Co., 1988.
- Smith, M. *The Mesoamerican Urban Landscape from Teotihuacan to the Aztecs*. Paper presented at the conference, "Archeology of Complex Societies: Centripetal and Centrifugal Forces," October 21, 1995, California State University, San Bernardino.
- Purdy, J. "Ancient Voices Speak Again in Carthage." *Research Reporter*. Vol. 25, No. 2 (Winter 1995). University of Georgia.

Website:

- www.ovpr.uga.edu/rcd/95w/carthage.html

Notes:



Figure This!

Math Challenges for Families

When DO most people

watch television?

Nielsen Ratings for Programs airing at 10:00 PM (EST), Nov. 20-26, 2000

Program	Day	Rating	Share
The Practice	Sunday	13.3	20.00
Monday Night Football	Monday	13.9	23.00
Judging Amy	Tuesday	11.6	19.00
Law and Order	Wednesday	12.2	21.00
ER	Thursday	15.2	27.00
Law and Order: SVU	Friday	9.8	18.00
The District	Saturday	9.8	17.00

[Excerpted from Nielsen Top 20 for the week of November 20-26, <<http://tv.yahoo.com/nielsen/>>]

Figure This! On which night of the week did the greatest number of US households watch the most popular television shows at 10:00 P.M.?

Hint: $\frac{\text{Rating}}{\text{Share}} \times 100$ = percent of TV owning households with TVs on watching a particular show at that time. How can you use the Nielsen data to determine the percentage of households watching television on a given night?

Making comparisons involves careful definitions and use of mathematics. Actuaries use comparisons to create insurance rates, biologists use this type of mathematics to compare different species in different eras, as do archaeologists for carbon dating artifacts.

Answer:
Sunday night.

Figure This!

Get Started:

In the Nielsen Company's rankings, *share* describes the percentage of active television sets on, and tuned to a particular program, out of all TV owning households, while *rating* describes the percentage of households who own televisions that are tuned to a particular show. A 15.2 rating means 15.2% of all households owning televisions were tuned to *ER*. A 27.0 share means that 27.0% of all those households with a television on at the time were tuned to *ER*. For *ER*,

$$\frac{\text{Rating}}{\text{Share}} = \frac{15.2}{27.0} \times 100 \approx 56$$

Thus, about 56% of the TV owning households had their TVs tuned to *ER*. What were the percentages for the other shows?

Complete Solution:

Using the relationship between the Nielsen rating and share in Getting Started, calculate the percentages for shows.

Program	Day	Rating	Share	% of TV owning Households With TV on
<i>The Practice</i>	Sunday	13.3	20.0	66.5%
<i>Monday Night Football</i>	Monday	13.9	23.0	60.4%
<i>Judging Amy</i>	Tuesday	11.6	19.0	61.1%
<i>Law and Order</i>	Wednesday	12.2	21.0	58.1%
<i>ER</i>	Thursday	15.2	27.0	56.3%
<i>Law and Order: SVU</i>	Friday	9.8	18.0	54.4%
<i>The District</i>	Saturday	9.8	17.0	57.6%

The highest percentage occurred on Sunday night.

Try This:

- Using a website, TV rating, or a newspaper, find the rating and share of your favorite TV show.
- Look on the web or in the library to determine how the Nielsen ratings are done.
- Check with your friends and neighbors to see if they have been surveyed for TV ratings.

Additional Challenges:

(Answers located in back of booklet)

1. For Thursday November 25, 2000, at 8:00 PM EST, *Friends* had a 9.6 rating and a 15.0 share. There were approximately 102.2 million TV owning households in the US at that time.
 - a. What percent of the households had their TV on during *Friends*?
 - b. Approximately how many households were watching *Friends*?

2. Companies that advertise on television are usually interested in the number of viewers of a certain age or gender. For example, adults from 18 to 49 are popular with many advertisers. In the United States, this group includes about 129 million people. On November 29, 2000, a *David Blaine Special* received a 7.3 rating and a 10.1 share among this group. If 1 rating point represents 1% of the target group, how many adults ages 18–49 watched this special? What percentage of the group were watching television at the time?
3. To compare the cost of advertising on two or more programs, advertisers use an index called the CPM (cost to reach 1000 households). The CPM is calculated as shown here:

$$\text{CPM} = \frac{\text{Cost per 30 second advertisement}}{\text{thousands of households watching the show}}$$

In 2000, the cost of 30-second ad on *ER* was \$600,000; its rating was 15.2. The cost of a 30-second ad on *Diagnosis: Murder* was \$51,000; its rating was 6.3. Compare the CPM for these two shows. (Each rating point represents 1,022,000 households.)

Things to Think About:

- What other types of entertainment are rated?
- Why are both rating and share important in the television industry?
- How is a song's popularity determined?
- Why do advertisers and networks care about the age and gender of viewers?
- How would you collect information on television-viewing habits?

Did You Know That?

- Nielsen Media Research measures audience levels using both electronic meters and diaries. The national sample is a combined sample of the "Nielsen families" using meters and diaries. It consists of about 5000 homes in 210 different television markets. The national Nielsen ratings also include key demographic information such as the number of adults ages 25–54 watching a particular show.
- The highest-rated TV show as of March 2001 was the final episode of *M*A*S*H*, which received a rating of 60.2 and a share of 77 on February 28, 1983.
- As of March 2001, 12 of the 25 highest-rated shows were Super Bowl broadcasts.
- In January 2001, the average number of televisions per US household was 2.4.
- About 60% of US teens have a television in their bedroom.
- In 1999, the automotive industry outspent all other industries for television advertising.

Resources:

Books:

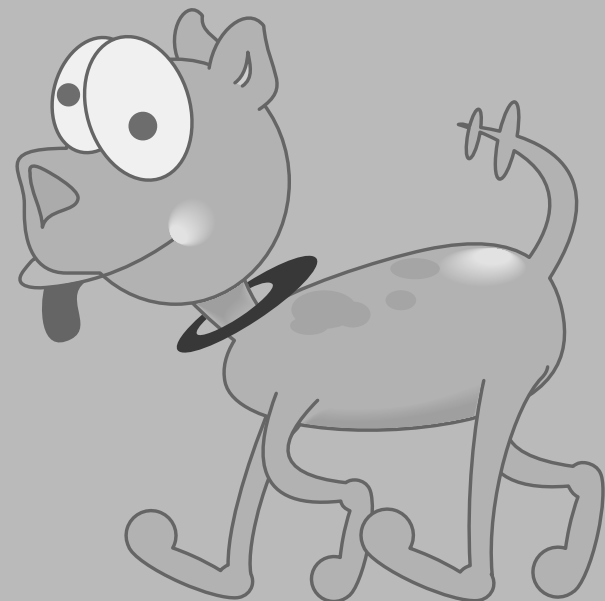
- Hall, Robert W. *Media Math*. Chicago, IL: NTC Business Books, 1991.
- *Consumer Reports*, January 2001.

Websites:

- www.nielsenmedia.com/whatratingsmean
- tv.yahoo.com/nielsen
- faculty.washington.edu/baldasty/Feb3.htm
- www.infoplease.com/ipsea/A0110473.html

Notes:

[illegible]





FigureThis!
Math Challenges for Families

How *Big* was the plate?

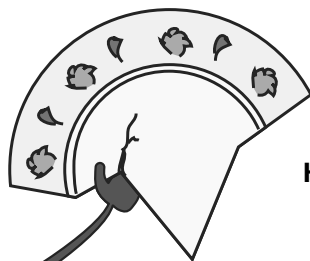
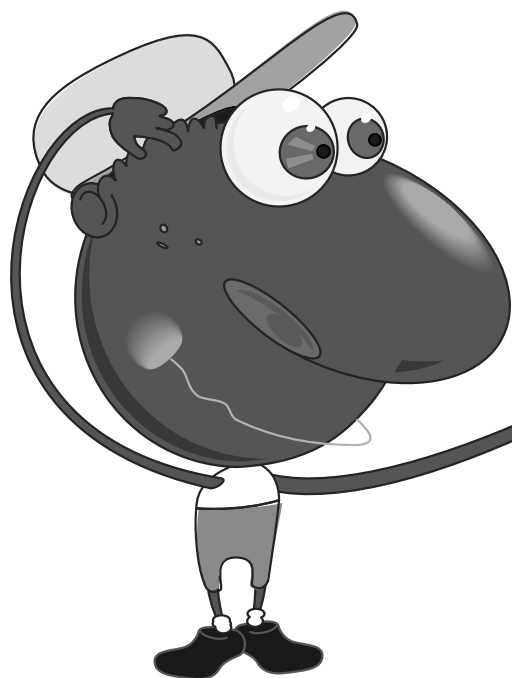
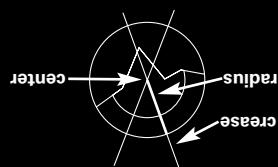


Figure This! While digging in an ancient city, an archeologist found this piece of a round plate.

How can Ratio estimate the size of the original plate?

Hint: How can you fold a circle to find its center?

Symmetry is a property of some mathematical shapes. Designers, architects, bakers, and decorators all use symmetry in their work.



Trace the rim of the piece of a plate on a sheet of paper. Fold the tracing of the rim onto itself to form a crease in the paper. Repeat this process using a different part of the arc. The point where the two creases intersect is the center of a circle. This circle is the same size as the original plate. (You can use a compass to complete the circle.)

Answer:

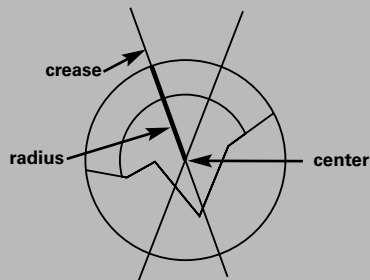
Figure This!

Get Started:

Trace the bottom of a glass to make a circle. Fold a circle of paper in half. Unfold it, then fold it in half again along a different line. What do you observe about the two creases?

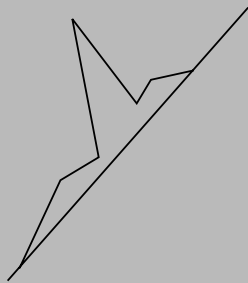
Complete Solution:

Trace the piece of a plate on a sheet of paper. Fold the tracing of the edge of the plate onto itself to form a crease in the paper. Repeat this process using a different part of the arc. The folds are along lines of symmetry for the circle that describes the shape of the original plate. These lines of symmetry contain diameters of the circle and meet at the center. The plate's radius is the distance from the center to the rim. Its diameter is twice the radius.



Try This:

- Sketch the figure below.



Fold the paper along the line, so that your drawing is on the outside, then trace the figure on the opposite side of the line. When you unfold the paper, the new image forms a figure with a line of symmetry.

- Look in the yellow pages of your local phone book for company logos that have lines of symmetry.
- Find a crossword puzzle and determine by folding if it has a line of symmetry.
- Draw a triangle. Through the midpoint of each side of the triangle, draw a line that forms 90° angles with the side (the perpendicular bisectors). These lines intersect at the center of a circle that contains each vertex of the triangle.

Additional Challenges:

(Answers located in back of booklet)

1. How many lines of symmetry does a rectangle have?
2. How could you find the center of a STOP sign?
3. What kinds of triangles have a line of symmetry?

Things to Think About:

- Does every figure have a center?
- What is a plane of symmetry?
- Why do museum curators restore ancient objects?
- How do paleontologists create a model dinosaur from fragments of fossilized bone?

Did You Know That?

- Many traditional Native American designs display symmetry.
- Scott Kim has developed a process of writing that involves "turn" symmetry (See the challenge "Upside Down.")
- Broken pieces or fragments of pottery are called potsherds, or shards.

Resources:

Books:

- Billstein, R., S. Libeskind, and J. Lott. *A Problem Solving Approach to Mathematics for Elementary Teachers*. Reading, MA: Addison Wesley Longman Publishing Co., 2000.
- Kim, Scott. *Inversions: A Catalog of Calligraphic Cartwheels*. Petersborough, NH: BYTE Books, 1981.
- Kim, Scott. *Alphabet Symmetry* (poster). Plains, NY: Cuisenaire/Dale Seymour Publications.

Website:

- www.scottkim.com



Figure This!

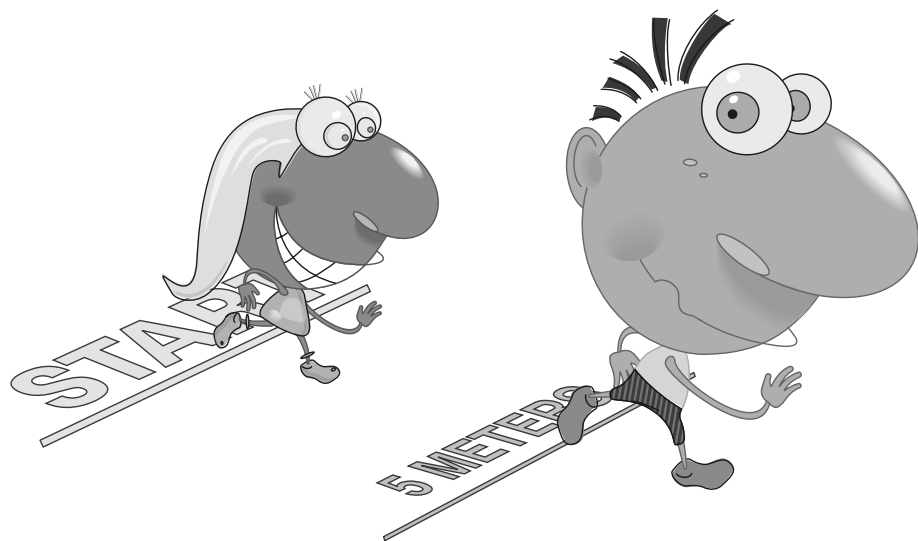
Math Challenges for Families

Who runs fffffaster?

Figure This! When Polygon and Exponent ran a 50-meter race, Polygon crossed the finish line while Exponent was at the 45-meter mark. The two friends decide to race again. This time, Exponent starts 5 meters ahead of Polygon, who is at the starting line. If each runs at the same speed as in the previous race, who will win?

Hint: Compare the distances traveled.

Mathematical problem solving requires logical reasoning. Lawyers, philosophers, electricians, and carpenters all use logical reasoning in their daily work.



The race will end in a tie.

Answer:

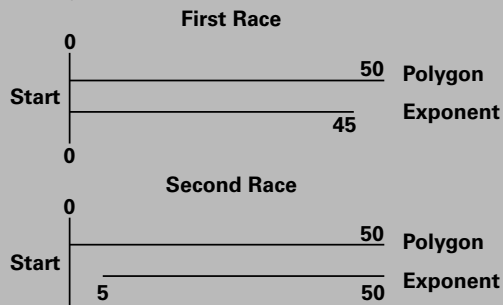
Figure This!

Get Started:

Draw a segment to represent each runner's position when the race is won. Draw another set of segments to represent the starting positions of the second race. What do you notice?

Complete Solution:

In the first race, Exponent ran 45 meters in the same time it took Polygon to run 50 meters. In the second race, the finish line is 45 meters from Exponent's starting position and 50 meters from Polygon's. Therefore, if each runs at the same speed as in the previous race, the race will end in a tie. See the diagram.



Try This:

- Run a race with a friend. After the race, decide on a fair handicap (the distance one of you gets to start ahead of the other faster person) for the winner, then race again. Who wins the second race?
- Look up the winning times for runners, Marion Jones (US) and Cathy Freeman (Australia), in the 2000 Olympics. How do these times compare with the world records for the distances they ran?

Additional Challenges:

(Answers located in back of booklet)

1. Polygon and Exponent decide to race a third time. In this race, Polygon starts 5 meters behind Exponent, who is at the starting line. If each runs at the same speed as before, who will win this time?
2. Imagine that Polygon and Exponent are running laps on a 50-meter oval. If each runs at the same speed as in previous races, after how many laps will Polygon be one whole lap ahead of Exponent?
3. If Exponent and Polygon start at opposite ends of the 50-meter track and run towards each other, where will they meet?

Things to Think About:

- Does the length of a race affect a runner's winning strategy?
- Why is chess called a game of logic?
- How are handicaps figured in golf? In what other sports are players allowed handicaps?

Did You Know That?

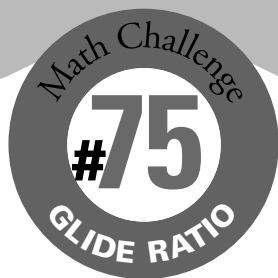
- The Greek mathematician Archimedes (ca. 287–212 B.C.) is sometimes considered the father of logic.
- Bertrand Russell (1872–1970), the English mathematician and philosopher, once said, "Mathematics is logic and logic is mathematics."
- Carpenters sometimes use a rope with knots spaced at 3, 7, and 12 units to make a right angle in a right triangle with sides 3, 4, and 5.
- There is an Association for Symbolic Logic.
- Former elementary teacher George Boole (1815–1864) developed an algebra of logic.

Resources:

Books:

- Gardner, M. *Goth: Paradoxes to Puzzle and Delight*. San Francisco, CA: W. H. Freeman and Co., 1982.
- House, P. (ed.). *Providing Opportunities for the Mathematically Gifted K-12* Reston, VA: National Council of Teachers of Mathematics, 1987.

Notes:



FigureThis!
Math Challenges for Families

How far can you **FLY** a *paper airplane?*

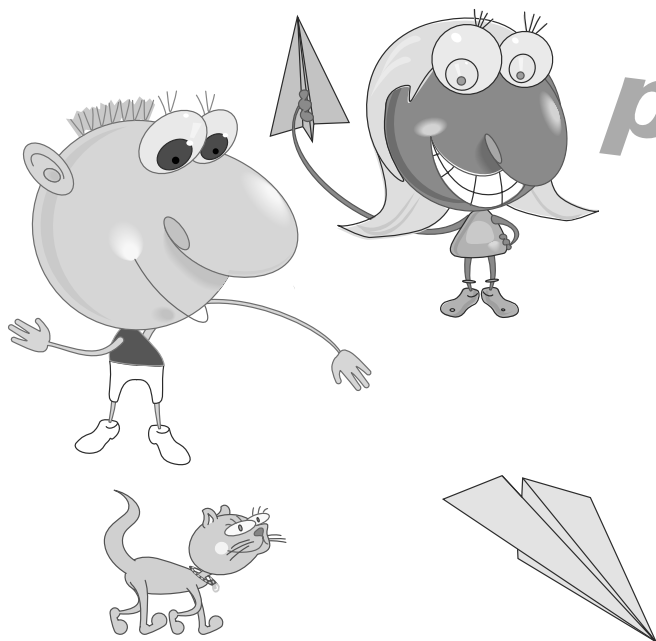


Figure This! If a glider has a glide ratio of $1/10$, it goes down 1 foot for every 10 feet it travels horizontally. Suppose two gliders start at the same height, one with a glide ratio of 0.3 and one with a glide ratio of $2/7$. If they glide until they hit the ground, which one will have glided the farther horizontal distance?

Hint: How can you compare the two glide ratios?

Comparing quantities expressed in different forms is critical in measurements, and in equations and rules. Mechanics repairing machines, computer programmers looking for efficient algorithms, and chemists working in laboratories have to find ways to compare quantities in different forms.

The glider with the glide ratio of $2/7$ will fly farther.

Answer:

Figure This!

Getting Started:

How far does each glider go by the time it has dropped 1 foot?
Draw a picture of the path of each glider.

Complete solution:

There are several ways to approach this challenge.

- One involves comparing how far the gliders descend after flying the same horizontal distance. To do this, write both ratios as fractions and find a common base (or denominator) for each one. For example, you could change both fractions to sevenths.

$$\text{A: Since } 0.3 = \frac{0.3}{1}, \text{ we have } 0.3 = \frac{7 \cdot 0.3}{7 \cdot 1} = \frac{2.1}{7}$$

$$\text{B: } \frac{2}{7}$$

One glider (A) descends 2.1 feet for every 7 feet of horizontal travel, while the other (B) falls only 2 feet over the same horizontal distance. The glider that falls more over the same horizontal distance will hit the ground. Therefore, glider B will fly farther.

- Another way to analyze this situation is to determine how far each glider has gone horizontally when both have dropped 1 foot. To do this, rewrite each glide ratio as its equivalent fraction with a numerator of 1.

$$\text{A: } 0.3 = \frac{0.3 \div 0.3}{1 \div 0.3} \approx \frac{1}{3.33}$$

and

$$\text{B: } \frac{2}{7} = \frac{2 \div 2}{7 \div 2} = \frac{1}{3.5}$$

This means that, after a 1-foot vertical drop, glider A travels about 3.3 feet, while B travels 3.5 feet. Therefore, glider B will fly farther.

Try This:

- Make two or more paper airplanes. Conduct a series of flight tests to estimate their glide ratios.
- Look up gliders and glide ratios on the web to find a discussion of glide ratios.

Additional Challenges:

(Answers located in back of booklet)

- Three gliders have glide ratios of 0.21, 7/42, and 14%, respectively. Which one will travel farthest?
- Three gliders have glide ratios expressed as 0.4, 1/2.5 and 0.04. Do any of these have a glide ratio of 2/5?
- A glider with a glide ratio of 2/9 has just cleared a line of trees 20 feet tall. About 20 feet farther along its flight path, there is a fence 6 feet tall. Will the glider make it over the fence?
- Polygon launches a glider with a glide ratio of 12% from the top of a 42-foot cliff. There is no wind. How far from the base of the cliff will the glider land? What is the actual distance traveled by the glider?

Things to Think About:

- What design elements might affect the glide ratio of a glider?
- Do gliders with greater glide ratios fly farther?
- Can a glider have a glide ratio of 0?
- How would you describe the glide ratio of a glider that drops straight down?

Did You Know That?

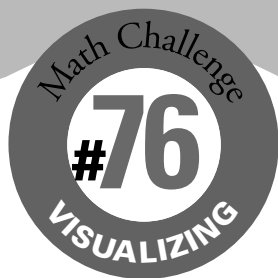
- The glide ratio can be determined by computing the tangent of the angle defined by the glider's path and the horizontal.
- An angle of depression is the angle from the horizontal looking down while the angle of elevation is the angle from the horizontal looking up.
- Some jets, such as the Harrier, can take off or land vertically.

Resources:

Books:

- De Lange, J. *Flying through Mathematics: Trigonometry, Vectors and Flying*. Scotts Valley, CA: Wings for Learning, 1991.

Notes:



FigureThis!
Math Challenges for Families

Is *seeing* believing?

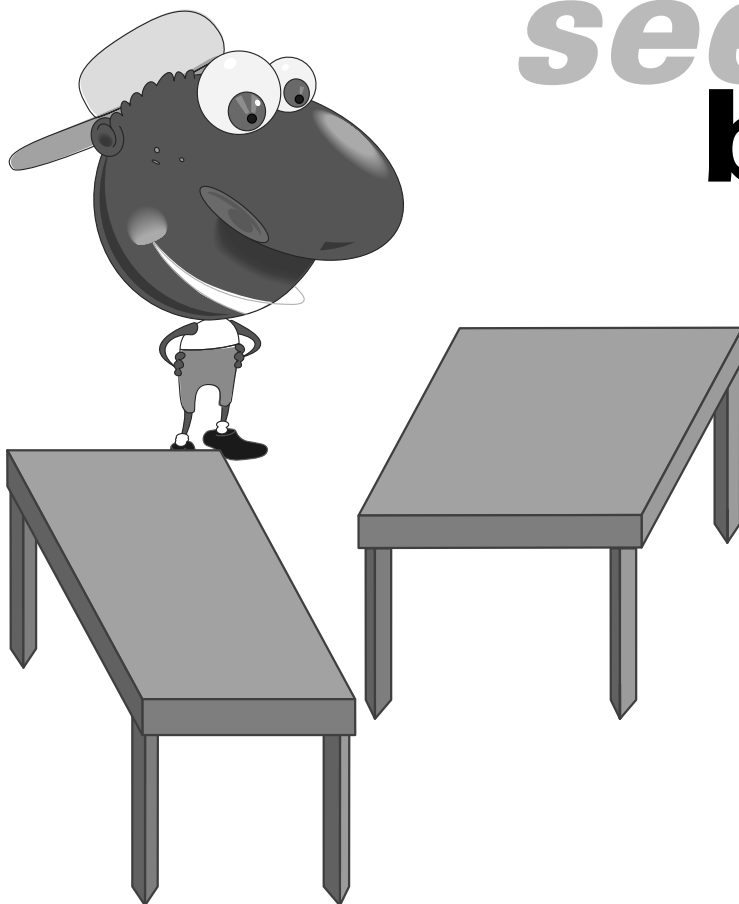


Figure This! Which table appears to have the larger top?

Hint: What do you have to know to compare the sizes?

Visualization is the ability to “see” an image in two- or three-dimensions. Visualization is important for architects, visual artists, and designers.

The tops have the same area.

Answer:

Figure This!

Getting Started:

Trace each top and compare them.

Complete solution:

- If you trace one top, it will exactly fit on the other. The objects may appear different because people visualize two- and three-dimensional objects differently.
- Another strategy is to measure and compute the actual area of each table top. Doing this will show that the tops have the same area.

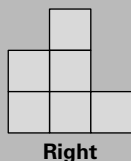
Try this:

- Walk around a chair or sofa in your home and notice how what you can see changes as you move.
- Using a sheet of grid paper, draw some letters of the alphabet so that they appear to be three-dimensional.
- The German artist Albrecht Dürer (1471–1528) created an engraving called “St. Jerome in his Cell.” Look in a book or on the web to find a reproduction of that engraving. In the engraving, look for lines that intersect in the picture but would be parallel in real life.

Additional Challenges:

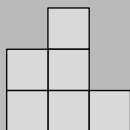
(Answers located in back of booklet)

1. The diagram below shows the view of a building from the right-hand side.



Right

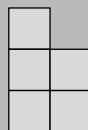
Given this fact, are there any of the views below that could not represent the front of the building?



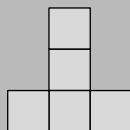
a



b



c

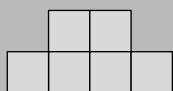


d

2. What is the maximum number of blocks that can be used to create the views of the structure whose front and right side views are pictured?



Front

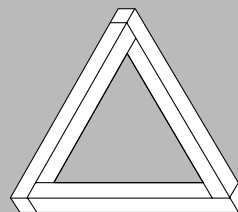


Right

3. Draw a cube. Then increase one dimension of the cube to draw a box that has three times the volume of the cube.

Things to Think About:

- A mirage is an imagined visualization in the desert.
- Many children's toys, such as Shape-O™, are designed to help children learn about visualization.
- Stick a pencil or straw into a glass of water. Does the object appear broken or bent? Why?
- Why does the moon look bigger when it is nearer the horizon, and smaller when it is higher in the sky?
- Could the triangular shape below be built of wood?



Did You Know That?

- The image that forms on your retina is flat, yet you see a world of shape, color, depth, and motion.
- Nature uses camouflage illusions that allows both animals and insects to match their surroundings so they are undetectable.
- The Poggendorf illusion, discovered in 1860, is one of the most famous distortion illusions known and uses colored lines passing through a rectangle.
- Mirage comes from the French word *mirer*, to look at; or *se mirer*, to be reflected. A mirage may include the appearance of lakes in deserts or on hot asphalt roads, the images of ships and icebergs (frequently seen as if inverted and suspended in the atmosphere of the Polar Regions), or “looming” as witnessed in mists or fogs.

Resources:

Books:

- Anno, Mitsumasa, *The Unique World of Mitsumasa Anno: Selected Illustrations 1969-1977*, Tokyo: Kodansha Ltd., 1977.
- Bergamini, D. *Life Science Library: Mathematics*. New York: Time Inc., 1963.
- Ernst, Bruno, *Adventures with Impossible Figures*. Norfolk, VA: Tarquin Publications, 1986.
- Ernst, Bruno, *The Magic Mirror of M. C. Escher*. Norfolk, VA: Tarquin Publications, 1985.
- McKim, R. *Experiences in Visual Thinking*. Monterey, CA: Brooks/Cole Publishing Co., 1972.

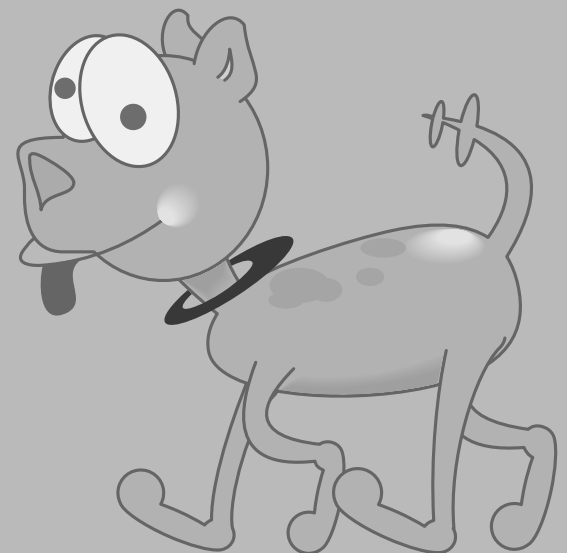
- Steinhaus, H. *Mathematical Snapshots*. New York: Oxford University Press, 1950.
- Turner, Harry. *Triad: Optical Illusions and How to Design Them*. New York: Dover Publications, 1978

Websites:

- www.crs4.it/Ars/arshtml/arstoc.html
- www.saumag.edu/art/studio/chalkboard/lp-intro.html
- www.sandlotscience.com/Camouflage/Camouflage_frm.htm

Notes:

Tangent



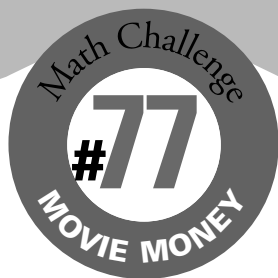


Figure This!

Math Challenges for Families

Do movies make \$ money?



Figure This! The theater box office receipts for the movie *Ratio in Magicville* for the past four weeks were \$15,000, \$12,000, \$12,000, and \$10,000, respectively. The theater owner pays the movie distributor the following percentages of the box-office receipts: 70% for each of the first two weeks, 60% for the third week, and 50% for the fourth week. Other operating expenses are \$4500 per week. Did the theater make or lose money?

Hint: How much did the theater have to pay the distributor the first week to show the movie?

Profit and loss calculations are essential to financing and operating any business. Business owners, shippers, accountants, and bankers all carefully examine income and expenses when making financial decisions.

After four weeks, the theater showed a loss of \$100.

Answer:

Figure This!

Get Started:

Make a table to keep track of the profit (or loss) each week.

Week	Box-Office Receipts	Percentage to Distributor	Amount to Distributor	Other Expenses	Profit (or Loss)
1	\$15,000				
2	\$12,000				
3	\$12,000				
4	\$10,000				
				Total	

Complete Solution:

For each week, subtract the distributor's share and other expenses from the theater box-office receipts. The result is the weekly profit (or loss). See the table. After four weeks, the theater has a loss of \$100.

Week	Box-Office Receipts, R	Percentage p , to Distributor	Amount, $A = Rp$ to Distributor	Other Expenses, E	Profit or (Loss) $R - A - E$
1	\$15,000	70%	\$10,500	\$4500	\$0
2	\$12,000	70%	\$8400	\$4500	-\$900
3	\$12,000	60%	\$7200	\$4500	\$300
4	\$10,000	50%	\$5000	\$4500	\$500
				Total	-\$100

Try This:

- Theater owners generally consider that it takes 10,000 ticket buyers to support one movie screen. Based on this rule, determine if your town has too many screens.
- Check your local newspaper for a report on box-office receipts, or consult an appropriate website such as www.film.com/reviews/rev_box/index.jhtml.
- Identify the movie that has had the longest run at your local theaters.
- Do some research in publications such as *Entertainment Weekly* to identify the box-office receipts made by your favorite movies.

Additional Challenges:

(Answers located in back of booklet)

1. Suppose the theater in the challenge extends the contract for *Ratio in Magicville*. For the next two weeks, the distributor's percentages will be 40% and 30%, respectively. If the theater hopes to earn a profit of at least \$1000 over the next two weeks, how much will have to be taken in at the box office?

2. For additional challenge 1, what would have to be the box office receipts for the profit to be \$500 for week 5 and \$500 for week 6?
3. A theater's typical profit for popcorn is around 85% of the price paid by the patron. If you paid \$4.00 for a bag of popcorn, about how much did it cost the theater owner?
4. A theater charges \$8.00 for admission. At the concession stand, a 32-ounce soda which costs \$0.30 to produce is sold for \$3.00. During the first four weeks of *Ratio in Magicville*, 40% of moviegoers bought sodas. About how much did the theater earn on soda?

Things to Think About:

- Why have many movie theater chains filed for bankruptcy in recent years?
- What is the most popular concession item sold at movies?
- Is it fair to compare box office receipts for movies such as *Titanic* and *Gone with the Wind*?
- What kind of movies have the longest box-office runs?

Did You Know That?

- Theater owners generally do not like long movies (three hours or more) because there's time for only one nightly showing per screen. This means fewer opportunities for concession sales and ticket sales.
- The bulbs for a movie projector cost about \$750 each.
- Movie theaters make 95% of their money on weekends.
- Only 20% of movies are highly profitable.
- In 1999, the United States had about 37,000 movie screens.
- As of May, 2001, the current leading money-maker was *Titanic*, which has grossed over \$600.8 million.

Resources:

Newspaper:

- *Dallas Morning News*. Nov. 26, 2000.

Websites:

- www.howstuffworks.com/movie-distribution3.htm
- www.hollywood.com/nato
- faculty.washington.edu/baldasty/200.htm
- www.film.com
- www.imdb.com
- www.film.com/reviews/rev_box/index.jhtml



FigureThis!
Math Challenges for Families

Does he have a chance?

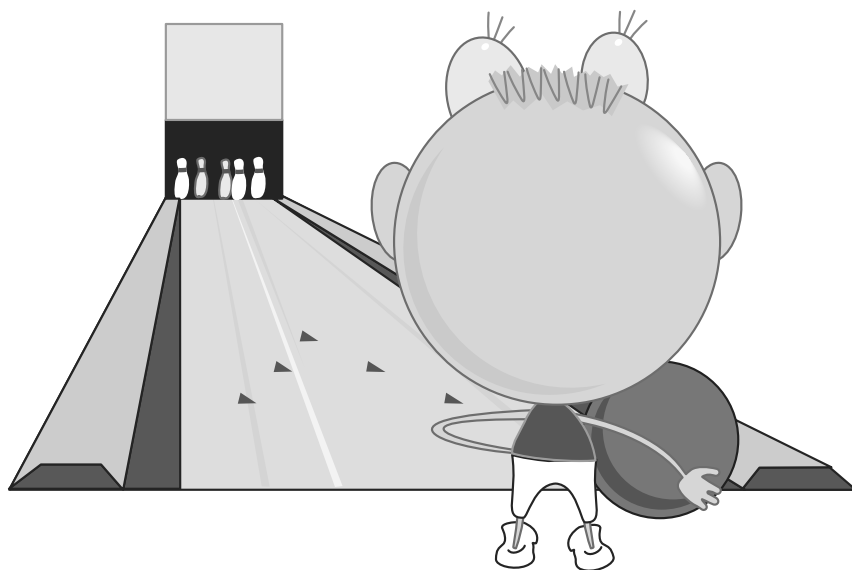


Figure This! Helix is about to roll his final game in a bowling tournament. His previous scores were 134, 99, 109, 117, and 101. To win the tournament, his average score must be at least 114. Considering his past performance, estimate his chances of winning.

Hint: What is the least score Helix needs to win the tournament?

Probabilities, or measures of chance, can be estimated considering the results of past events. People who calculate insurance premiums, batting averages, and medical treatment risks all use past events to estimate probabilities.

Figure This!

Get Started:

How many games will Helix bowl in the tournament? To reach an average of at least 114, what is the lowest possible sum of his scores?

Complete Solution:

- Helix will bowl a total of six games. To reach an average of at least 114, his total score must be at least 6×114 , or 684. The sum of his scores after five games is 560. Subtracting 560 from 684 leaves 124. Therefore, any score 124 or higher will result in a win for Helix. Now, consider that Helix has scored higher than 124 in only one of the previous five games. Based on these performances, you might estimate that his chances of winning are $1/5$, or 20%.
- Another way to figure the average is to subtract 114 from each score and add the differences. In order, his scores are +20, -15, -5, +3, -13 from 114.

The total is $+20 + (-15) + (-5) + 3 + (-13) = -10$

He must score at least 10 above the desired average of 114 to compensate, or 124, again having $1/5$ chance, or 20%.

Try This:

- Toss a thumbtack into the air 200 times and record how it lands, point up or point down. Use your results to estimate the probability that a thumbtack will land with its point up.
- Look at the test scores you presently have in one of your classes and the grade scale for your school. Determine if it is possible to have a B average in the class after your next test.

Additional Challenges:

(Answers located in back of booklet)

1. The average of 5 test scores is 84%. If the highest grade is 100%, what is the lowest possible score for one of these tests?
2. In diving contests, the highest and lowest scores from the individual judges are dropped before calculation of the final average. One diver received the following scores: 9.6, 9.4, 9.7, 9.8, 9.8, and 9.9. In this case, does the practice of dropping the highest and lowest scores help or hurt the final average?
3. In a basketball contest, 80% of a player's free throws had been made after 130 shots. If the player makes the next 5 shots, what is the percentage made?

Things to Think About:

- How are handicaps determined for bowlers?
- An average (or mean) may be affected by one very high or one very low number.

- Is the average always the best way to describe the "center" of a set of data?
- In many cases, the results from a small number of experiments will not provide a very accurate estimate of the probabilities involved.

Did You Know That?

- The highest individual bowling score for three consecutive games is 900 by Jeremy Sonnenfeld of Lincoln, Nebraska, on February 2, 1997. These are perfect games of rolling only strikes (knocking all ten pins down in one roll); the record cannot be broken.
- During World War II, John Kerrich, a prisoner of war, tossed a coin 10,000 times, recording 5067 heads.
- In 1777, Georges Louis Leclerc, Comte de Buffon, described the first example of a geometrical probability. He used a needle and a lined sheet of paper to approximate a value for pi (π).
- Probability was used to show that the selection of draftees for the Vietnam War was biased against certain groups in the country, in that they were overrepresented among the forces sent to Vietnam.

Resources:

Books:

- Eves, Howard. *An Introduction to the History of Mathematics*. Philadelphia, PA: Saunders Publishing Co., 1983.
- *The Guinness Book of World Records*. New York: Bantam Books, 1999.

Notes:



FigureThis!
Math Challenges for Families

Can you
turn a frown
into a smile?

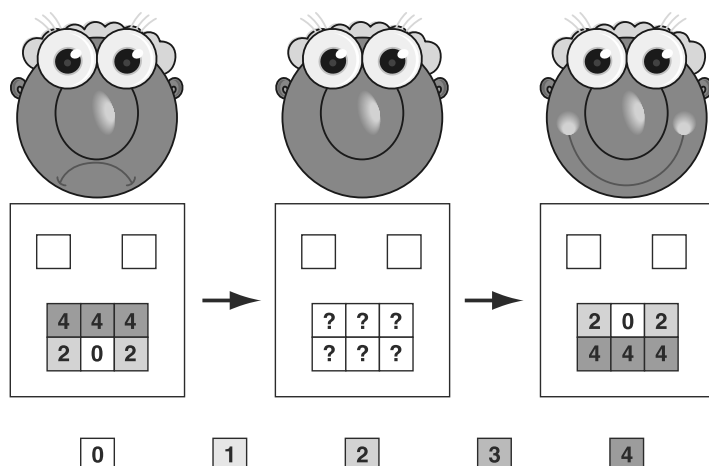
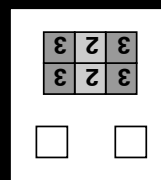
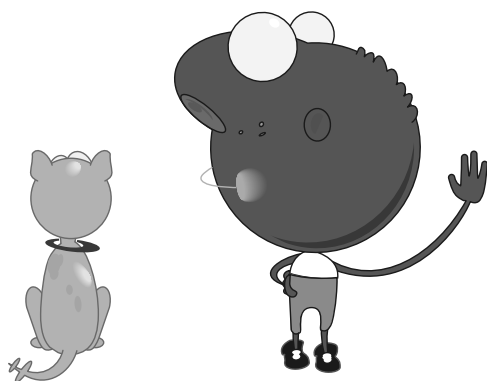


Figure This! In computer graphics, morphing can be used to gradually change one image to another. A frown can be "morphed" to a smile by changing the shades of the squares. As you morph the frown on the left to the smile on the right, what shades should be in the image in the middle?

Hint: Use only the numbers 0, 1, 2, 3, and 4 to represent the five different shades. What shade is halfway between 0 and 4?

Pictures or sounds can be "digitized" using a collection of numbers. Digitized images or sounds can be morphed or manipulated by arithmetic. Special effects in movies, pictures on computers, and sounds on some CDs use morphing.



One answer is shown here.

Answer:

Figure This!

Get Started:

Replace each shade in the graphics by the corresponding number. Look at the grids of numbers and decide what “halfway” might mean.

Complete Solution:

Replaces each shade in the frown and the smile by the corresponding number, as shown here:

4	4	4
2	0	2

?	?	?
?	?	?

2	0	2
4	4	4

To find the appropriate numbers for each square in the middle image, average the numbers in the matching squares of the frown and the smile. The grid on the right below shows the appropriate numbers while the grid on the left shows the corresponding shades.

3	2	3
3	2	3

3	2	3
3	2	3

To create the illusion of a smooth transition, an actual morphing uses many more shades and has many more steps.

Try This:

- Design a set of flip cards so that an image changes shape gradually from one card to the next and so that when you “flip” through the cards, the image appears to move.
- Design grid patterns to represent the letters A and B. If you morphed the A to the B, what would the half-way image look like?
- Search the web for sites that feature morphed images.
- Ask your art or computer teacher if your school has software capable of morphing images.

Additional Challenges:

(Answers located in back of booklet)

1. Draw the grid that represents an image which is 70% transformed from the frown to the smile.
2. The town of Belgrade is halfway between Bozeman and Butte and in a line with them. Bozeman is located at mile marker 200, while Belgrade is at mile marker 155. At what mile marker is Butte?
3. A boat is sailing in a line from Miami to Bermuda. Miami is about 1000 miles south and 100 miles west of New York City. Bermuda is about 400 miles south and 300 miles east of New York City. What is the boat's location, in relation to Miami, when it has completed 30% of its trip?

Things to Think About:

- As noted in the solution to the challenge, an actual morphing uses many more shades (or colors) and has many more steps. How would you make an image appear to morph more slowly at the beginning and faster at the end?

- How do police “identi-kits” use morphing?
- Most computer images consist of colors, not just shades of black and white. How would you assign numbers to different colors?

Did You Know That?

- The musical equivalent to morphing is the *segue*.
- The dimensions of a typical computer screen are approximately 800 by 600 squares, or pixels. Each of these almost one-half million pixels may display one of thousands of different colors and shades.
- One step in the morphing of a full-screen image can be completed in less than 0.1 seconds.
- At a typical movie theater, viewers are shown 24 different frames (or images) per second.
- To create the movie *Toy Story II*, animators at Pixar Studios drew about 4000 key storyboard sketches by hand. To create frames for these key sketches, they used computers to manipulate elements on the screen. After that, they relied on software to fill screens using variations of morphing.
- In a style of painting known as *pointillism*, painters apply dots of paint to the canvas to create images. The French artist Georges Seurat (1859–1891) is perhaps the best-known artist who used this style.
- *Time Magazine* in the fall of 1993 published an article entitled “The New Face of America: How Immigrants Are Shaping the World's First Multicultural Society.” The cover of that issue featured a “morphed” image of a woman using various racial and ethnic features over the caption “The New Face of America.”

Resources:

Magazine:

- Porter, T., and G. Susman. “Creating Lifelike Characters in Pixar Movies.” *Communications of the ACM* 43 (No. 1, 2000): 25–29.
- Liss, Steve. “The New Face of America: How Immigrants Are Shaping the World's First Multicultural Society.” *Time* 142 (Fall 1993): 3–7.

Websites:

- hera.itc.it:3003/list_project.html#spotit
- www.chilhavisto.rai.it/Clv/English/mele.htm
- odyssey.ucc.ie/www/misc/model1.html
- www.publish.csiro.au/cyberscience/helix/TH60/TH60A1.htm
- www.pixar.com
- www.toystory2.com
- www.siggraph.org
- www.awn.com



FigureThis!
Math Challenges for Families

Is there room for all these gifts?



Figure This! Exponent has won the grand prize in a radio sweepstakes. His gifts will be delivered over 10 days. On the first day, he gets a kitten. On the second day, he gets two hummingbirds and a kitten. On the third day, he receives three rings, two hummingbirds, and a kitten. This pattern continues for seven more days! The remaining gifts, in order of appearance, include books, boxes of candy, bracelets, bouquets, cologne, CDs, and puppies. After 10 days, which gift did he receive most?

Hint: How many of each gift did Exponent receive on each day? Make a table to organize this information.

Organizing information and making decisions based on the information are necessary to solve problems. Census takers, librarians, managers, and demographers have methods to organize large quantities of information.

The most numerous gifts are boxes of candy and bracelets (30 each).

Answer:

Figure This!

Get Started:

One way to approach this problem is to list the gifts received on each day. This information could be organized in a table like the one below.

Day	1	2	3	4	5	6	7	8	9	10	Total
Kitten	1	1	1								
H-Bird		2	2								
Ring			3								
Book											
Candy											
Bracelet											
Bouquet											
Perfume											
CD											
Puppy											

Complete Solution:

Use the pattern described in the challenge to complete the table of gifts received on each day. Then calculate the total number of each type of gift. As shown below, Exponent receives a total of 30 boxes of candy and 30 bracelets.

Day	1	2	3	4	5	6	7	8	9	10	Total
Kitten	1	1	1	1	1	1	1	1	1	1	10
H-Bird		2	2	2	2	2	2	2	2	2	18
Ring			3	3	3	3	3	3	3	3	24
Book				4	4	4	4	4	4	4	28
Candy					5	5	5	5	5	5	30
Bracelet						6	6	6	6	6	30
Bouquet							7	7	7	7	28
Perfume								8	8	8	24
CD									9	9	18
Puppy										10	10

Try This:

- Build a model staircase using sugar cubes (or other small blocks). How many cubes does it take to build a 12-step staircase?
- Ask your local librarian what kinds of books are most popular in the library and how the decision was made.

Additional Challenges:

(Answers located in back of booklet)

1. What is the total number of gifts received in the challenge?
2. Find the solution to this English nursery rhyme.

As I was going to St. Ives,

I met a man with 7 wives.

Each wife had 7 sacks.

Each sack had 7 cats.

Each cat had seven kits.

Kits, cats, sacks, and wives,

How many were going to St. Ives?

3. If each of the 25 people in a room shakes the hand of each of the other people in the room and no person shakes any other person's hand twice, how many handshakes take place?
4. What would happen if the order of the gifts in the challenge were reversed? In other words, suppose Exponent received a puppy on the first day, two CDs and a puppy on the second day, and so on. Would your solution be the same?

Things to Think About:

- How could you determine the total value of the gifts described in the challenge?
- How many gifts would be received if the pattern continued for 365 days?
- How does the counting in this challenge compare to the counting in Challenge #7 "Double or Not"?

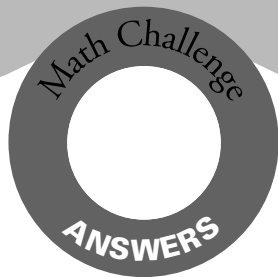
Did You Know That?

- The numbers of gifts received on successive days form this pattern: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55. These are triangular numbers. The total handshakes for increasing numbers of people also are triangular numbers, as are the total possibilities for double-dip ice cream cones for increasing numbers of flavors.
- A sequence like 4, 7, 10, 13, ..., in which you add 3 each time is an example of an arithmetic sequence.
- A sequence like 1, 5, 25, 125, ..., in which you multiply by 5 each time is an example of a geometric sequence.

Books:

- Notes:**

A cartoon illustration of a grey dog with large, expressive eyes, a black collar, and a small tuft of hair on its back, standing on a light grey background.



FigureThis!
Math Challenges for Families

Looking for answers?

Here are the answers for the
Additional Challenges section
of each Challenge.

Figure This!

Challenge 65

1. The answer may vary. The pattern shows that the factors to go from one complexion type to another are 3, 2.5, and 2. With this pattern, the next factor is 1.5 giving 45 as the recommended SPF.
2. The potato will be done in about 24 more minutes.
3. About 17 minutes.

Challenge 66

1. The ends of one runway should be labeled 26 and 8; the others should be 35 and 17.
2. The ends of one runway should be labeled 13 and 31; the ends of the other should be 4 and 22. The ends of the north-south runway are 18 and 36.
3. 190° .

Challenge 67

1. Probably, since 10° C is 50° F .
2. 30° C .
3. About 67° F .
4. -40° .
5. No.

Challenge 68

1. Graph a.

2a.

	Liked <i>Titanic</i>	Disliked <i>Titanic</i>	Totals
Liked <i>Star Wars</i>	70	43	113
Disliked <i>Star Wars</i>	50	37	87
Totals	120	80	200

- 2b. For the people in this study, there does not seem to be a strong association between liking one movie and liking the other. About 58%—a little more than half ($70/120$)—of those who liked *Titanic* also liked *Star Wars, Part IV*.
3. The graph reveals little connection between exercise and computer use. Most people in the study frequently use computers, whether they exercise or not.

Challenge 69

1. $1/2$.
2. 0.
3. 100% or 1.
4. less than.
5. $4/6$ or $2/3$

Challenge 70

- 1a. Toyota Prius; it looks like city mpg and highway mpg were reversed.
- 1b. Greatest change is either the Chevrolet Impala or the Saturn SW at 11 mpg, least is Toyota Tacoma at 3 mpg.
- 2a. The equation predicts a highway mpg of about 18.89 or about 19.
- 2b. The prediction is not far off from the actual mileage.
3. Answers may vary, depending on the relative value assigned to field-goal percentage and rebounds. One approach identifies the highest displayed value in each category and uses it as an “ideal.” In this case, the ideal point would be (0.574, 1157). If players are ranked according to this ideal, they fall in the following order: O’Neal, Mutombo, Mourning, Robinson, Divac, Davis, and Ratliff.

Challenge 71

1. The parts are symmetric because the sum of the differences between two sites is the same whether you compare A to B or B to A.
2. The population density of Minnesota is about 60 people per square mile, or about 23 people per square kilometer. This means that the population density in the study area was about 9 times as great.
3. The prices at stores A and E are the most similar, as their sum of differences is only \$0.64. The prices at stores C and E are the most different, with a sum of \$2.38.

Challenge 72

- 1a. 64%
- 1b. 9.8 million
2. About 9.42 million adults ages 18 – 49 watched the show. Of the target group, 72.3% were watching television.
3. The CPM for *ER* was \$38.62; the CPM for *Diagnosis: Murder* was \$792.

Challenge 73

- 1. If the rectangle also is a square, then it has four lines of symmetry. If not, then it has two, neither of which is a diagonal.
- 2. Find two lines of symmetry. They will intersect in the center.
- 3. A triangle with at least two sides of the same length (an isosceles triangle).

Challenge 74

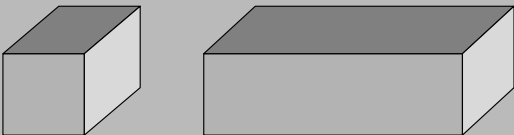
- 1. Polygon will win this race. Although they will be tied at the 45-meter mark, Polygon will run the last 5 meters faster than Exponent.
- 2. At their previous speeds, Polygon will run 500 meters (10 laps) in the same time that Exponent runs 450 meters (9 laps). Therefore, after 10 laps, Polygon will be 1 lap ahead of Exponent.
- 3. They will meet about 26.3 meters from Polygon's starting point (or 23.7 meters from Exponent's starting point).

Challenge 75

- 1. The one with the glide ratio of 14%.
- 2. The ratios 0.4 and 1/2.5 are both the same as 2/5.
- 3. Yes.
- 4. The glider will land at a horizontal distance of about 350 feet from the base of the cliff. Its actual flight path is about 353 feet long.

Challenge 76

- 1. No
- 2. 20 maximum
- 3. Answers will vary. One possible solution is shown here:



Challenge 77

- 1. Answers may vary. For example, box-office receipts of \$9000 in week 5 and \$6600 in week 6 will yield a profit of \$1020.
- 2. About \$8333 for week 5 and about \$7142 for week 6.
- 3. \$0.60.
- 4. About \$6615.

Challenge 78

- 1. 20%.
- 2. It helps. The average of the six scores is 9.7. After dropping the high and low scores, the average is 9.725.
- 3. About 81%.

Challenge 79

- 1. To create an image that is 70% of the way to the smile, you must identify the number that is 70% (or 0.7) of the way from each number on the smile to its corresponding number on the frown. For example, the number 4 on the frown corresponds to 2 on the smile. The difference of 4 and 2 is 2. Find 70% of this difference, $0.7(2) = 1.4$. Since $4 - 1.4 = 2.6$, the corresponding square on the image should be 3 (rounding to the nearest whole number).

3	1	3
3	3	3

3	1	3
3	3	3

- 2. 110.
- 3. It will be 180 miles north and 120 miles east of Miami.

Challenge 80

- 1. 220.
- 2. 1 (The man with the wives was going the other way.)
- 3. 300.
- 4. Yes.